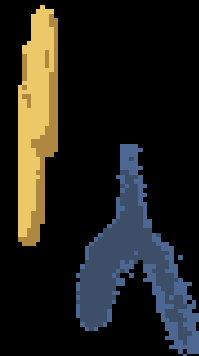


# Holography, Quantum Gravity and Black Holes

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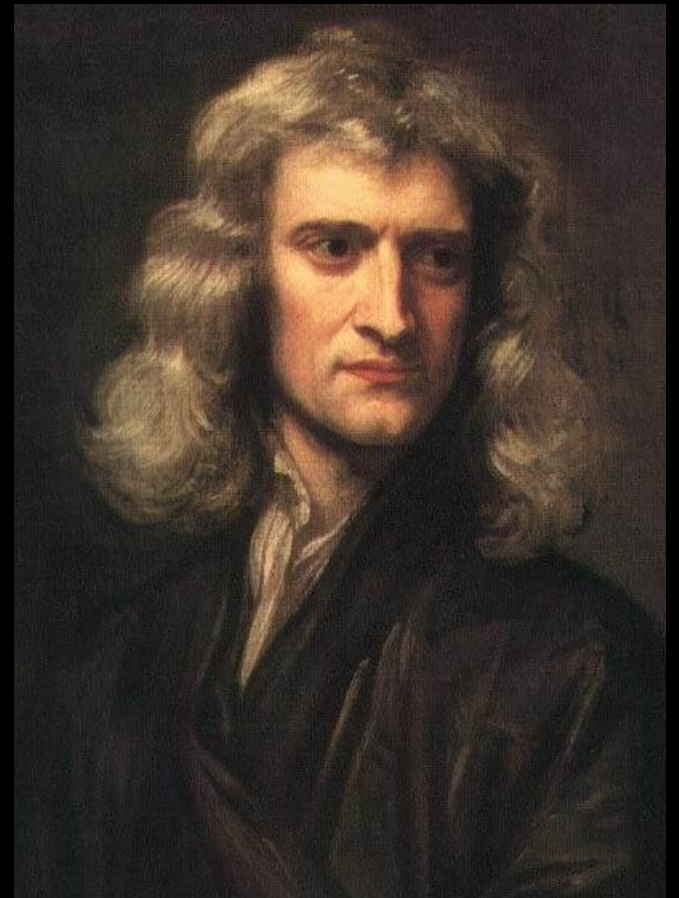
Jan de Boer  
Universiteit van Amsterdam

Nordita/AlbaNova  
18 February 2010



Law of Nature:

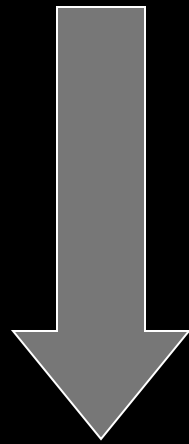
$$F = ma$$



Laws of Nature explain **how** the world works and allow one to **predict** the outcome of experiments.

Laws of Nature do not explain **why** the world is the way it.

Classical mechanics a la Newton:  
 $c = \text{infinite}$



Good  
approximation at  
low velocities.

Classical mechanics a la Einstein:  
 $c = \text{finite}$  (special relativity)

clip (© Marc Borchers)

# THE NEXT STEP

Classical mechanics  
(particle  $\neq$  wave)



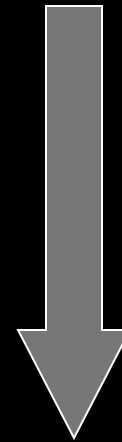
Special relativity  
(particle  $\neq$  wave)

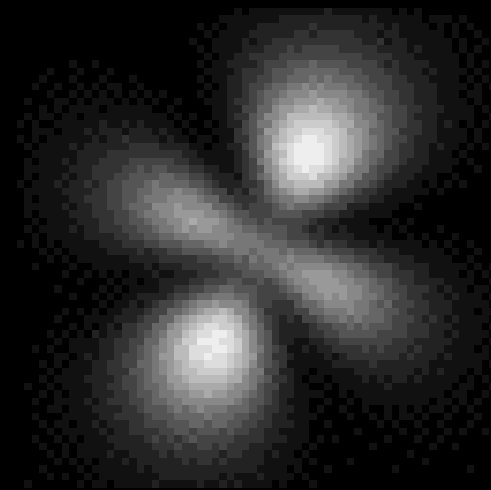
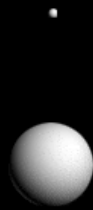


Quantum mechanics  
(particle=wave)



Quantum Field Theory  
(particle=wave)





particle  $\neq$  wave

particle = wave

The **standard model** describes all fundamental particles and forces that we know, except gravity.

This is an example of a quantum field theory.

Having an equation does not imply that it is easy to compute everything.

The standard model contains many free parameters.

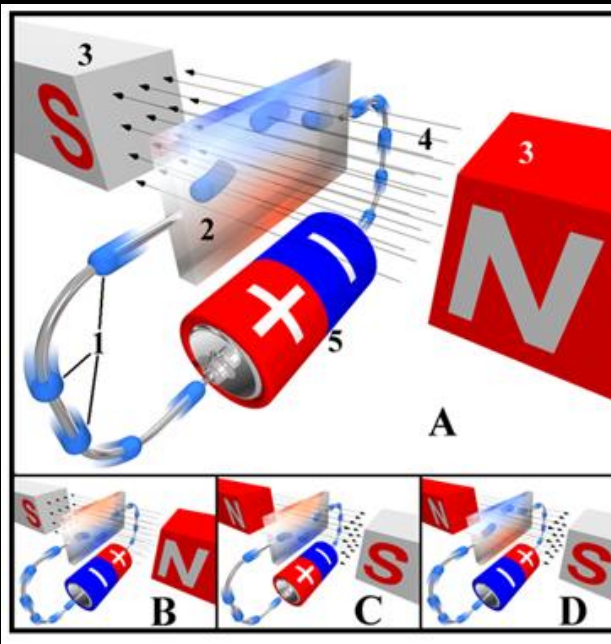
$$\begin{aligned}
 & -\frac{1}{2}\partial_\nu g_\mu^\alpha \partial_\nu g_\mu^\alpha - g_\alpha \int^{abc} \partial_\mu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4}g_\alpha \int^{abcd} \partial_\mu g_\mu^a \partial_\nu g_\mu^b \partial_\nu g_\mu^c \partial_\mu g_\mu^d + \\
 & \frac{1}{2}g_\alpha^2 (g_\mu^\alpha \gamma^\mu g_\mu^\alpha) g_\mu^\alpha + G^\alpha \partial^\alpha G^\alpha + g_\alpha \int^{abc} \partial_\mu G^\alpha G^\alpha g_\mu^\alpha - \partial_\nu W_\mu^\alpha \partial_\nu W_\mu^\alpha - \\
 & M^2 W_\mu^\alpha W_\mu^\alpha - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
 & \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^\dagger \partial_\mu \phi - M^2 \phi^\dagger \phi - \frac{1}{2}\partial_\mu \phi^\dagger \partial_\mu \phi^0 - \frac{1}{2}M \phi^\dagger \phi^0 - \beta_h \left[ \frac{2M}{\phi^0} + \right. \\
 & \left. \frac{2M}{\phi} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^\dagger \phi) \right] + \frac{2M}{\phi^0} \alpha_h - i g_{\nu\omega} [\partial_\nu Z_\mu^0 (W_\mu^\nu W_\nu^\omega - \\
 & W_\nu^\omega W_\mu^\nu) - Z_\nu^0 (W_\mu^\nu \partial_\nu W_\mu^\omega - W_\mu^\omega \partial_\nu W_\nu^\nu) + Z_\nu^0 (W_\nu^\omega \partial_\nu W_\mu^\omega - \\
 & W_\nu^\omega \partial_\nu W_\mu^\nu)] - i g_{\nu\omega} [\partial_\nu A_\mu (W_\mu^\nu W_\nu^\omega - W_\nu^\omega W_\mu^\nu) - A_\nu (W_\mu^\nu \partial_\nu W_\mu^\omega - \\
 & W_\mu^\omega \partial_\nu W_\nu^\nu) + A_\nu (W_\nu^\omega \partial_\nu W_\mu^\omega - W_\nu^\omega \partial_\nu W_\mu^\nu)] - \frac{1}{2}g^2 W_\mu^\alpha W_\mu^\alpha W_\nu^\beta W_\nu^\beta + \\
 & \frac{1}{2}g^2 W_\mu^\alpha W_\nu^\beta W_\mu^\beta W_\nu^\alpha + g^2 \alpha_w^2 (Z_\mu^0 W_\mu^\alpha Z_\nu^0 W_\nu^\alpha - Z_\mu^0 Z_\nu^0 W_\mu^\alpha W_\nu^\alpha) + \\
 & g^2 \alpha_w^2 (A_\mu W_\mu^\alpha A_\nu W_\nu^\alpha - A_\mu A_\nu W_\mu^\alpha W_\nu^\alpha) + g^2 \alpha_w \alpha_h [A_\mu Z_\mu^0 (W_\mu^\alpha W_\nu^\alpha - \\
 & W_\nu^\alpha W_\mu^\alpha) - 2A_\mu Z_\mu^0 W_\nu^\alpha W_\nu^\alpha] - g_{\nu\alpha} [H^2 + H\phi^0 \phi^0 + 2H\phi^\dagger \phi] - \\
 & \frac{1}{8}g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^\dagger \phi)^2 + 4(\phi^0)^2 \phi^\dagger \phi + 4H^2 \phi^\dagger \phi + 2(\phi^0)^2 H^2] - \\
 & g M W_\mu^\alpha W_\mu^\alpha H - \frac{1}{2}g \frac{M}{\alpha_w} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}i g [W_\mu^\alpha (\phi^0 \partial_\mu \phi - \phi \partial_\mu \phi^0) - \\
 & W_\mu^\alpha (\phi^0 \partial_\mu \phi^\dagger - \phi^\dagger \partial_\mu \phi^0)] + \frac{1}{2}g [W_\mu^\alpha (H \partial_\mu \phi - \phi \partial_\mu H) - W_\mu^\alpha (H \partial_\mu \phi^\dagger - \\
 & \phi^\dagger \partial_\mu H)] + \frac{1}{2}g \frac{1}{\alpha_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - i g \frac{\alpha_w}{\alpha_h} M Z_\mu^0 (W_\mu^\alpha \phi - W_\mu^\alpha \phi^\dagger) + \\
 & i g_{\nu\omega} M A_\mu (W_\mu^\nu \phi - W_\mu^\nu \phi^\dagger) - i g \frac{1}{2\alpha_w} Z_\mu^0 (\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger) + \\
 & i g_{\nu\omega} A_\mu (\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger) - \frac{1}{4}g^2 W_\mu^\alpha W_\mu^\alpha [H^2 + (\phi^0)^2 + 2\phi^\dagger \phi] - \\
 & \frac{1}{4}g^2 \frac{1}{\alpha_w} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2\alpha_w^2 - 1)\phi^\dagger \phi] - \frac{1}{2}g^2 \frac{\alpha_w}{\alpha_h} Z_\mu^0 \phi^0 (W_\mu^\alpha \phi + \\
 & W_\mu^\alpha \phi^\dagger) - \frac{1}{2}i g^2 \frac{\alpha_w}{\alpha_h} Z_\mu^0 H (W_\mu^\alpha \phi - W_\mu^\alpha \phi^\dagger) + \frac{1}{2}g^2 \alpha_w A_\mu \phi^0 (W_\mu^\alpha \phi + \\
 & W_\mu^\alpha \phi^\dagger) + \frac{1}{2}i g^2 \alpha_w A_\mu H (W_\mu^\alpha \phi - W_\mu^\alpha \phi^\dagger) - g^2 \frac{\alpha_w}{\alpha_h} (2\alpha_w^2 - 1) Z_\mu^0 A_\mu \phi^\dagger \phi - \\
 & g^2 \alpha_w^2 A_\mu A_\mu \phi^\dagger \phi - e^\lambda (\gamma^\partial + m_\lambda^2) e^\lambda - \partial^\lambda \gamma^\partial u^\lambda - \bar{e}_\lambda^\dagger (\gamma^\partial + m_\lambda^2) \bar{e}_\lambda^\dagger - \\
 & \bar{d}_\lambda^\dagger (\gamma^\partial + m_\lambda^2) \bar{d}_\lambda^\dagger + i g_{\nu\omega} A_\nu [-(e^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{e}_\lambda^\dagger \gamma^\mu \bar{e}_\lambda^\dagger) - \frac{1}{3}(\bar{d}_\lambda^\dagger \gamma^\mu \bar{d}_\lambda^\dagger)] + \\
 & \frac{5\alpha_w}{4\alpha_h} Z_\mu^0 [(e^\lambda \gamma^\mu (1 + \gamma^5) u^\lambda) + (e^\lambda \gamma^\mu (4\alpha_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{e}_\lambda^\dagger \gamma^\mu (\frac{4}{3}\alpha_w^2 - \\
 & 1 - \gamma^5) \bar{e}_\lambda^\dagger) + (\bar{d}_\lambda^\dagger \gamma^\mu (1 - \frac{8}{3}\alpha_w^2 - \gamma^5) \bar{d}_\lambda^\dagger)] + \frac{5\alpha_w}{2\sqrt{2}} W_\mu^\alpha [(e^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + \\
 & (\bar{e}_\lambda^\dagger \gamma^\mu (1 + \gamma^5) C_{\lambda\alpha} \bar{d}_\lambda^\dagger) + \frac{5\alpha_w}{2\sqrt{2}} W_\mu^\alpha [(e^\lambda \gamma^\mu (1 + \gamma^5) u^\lambda) + (\bar{d}_\lambda^\dagger C_{\lambda\alpha}^\dagger \gamma^\mu (1 + \\
 & \gamma^5) \bar{e}_\lambda^\dagger)] + \frac{5\alpha_w}{2\sqrt{2}} \frac{m_\lambda^2}{M^2} [-\phi^\dagger (\partial^\lambda (1 - \gamma^5) e^\lambda) + \phi (e^\lambda (1 + \gamma^5) u^\lambda)] - \\
 & \frac{8}{3} \frac{m_\lambda^2}{M^2} [H (e^\lambda e^\lambda) + \bar{e}_\lambda^\dagger (e^\lambda \gamma^5 e^\lambda)] + \frac{5\alpha_w}{2\sqrt{2}} \phi^\dagger [-m_\lambda^2 (\bar{e}_\lambda^\dagger C_{\lambda\alpha} (1 - \gamma^5) \bar{d}_\lambda^\dagger) + \\
 & m_\lambda^2 (\bar{e}_\lambda^\dagger C_{\lambda\alpha} (1 + \gamma^5) \bar{d}_\lambda^\dagger) + \frac{5\alpha_w}{2\sqrt{2}} \phi [m_\lambda^2 (\bar{d}_\lambda^\dagger C_{\lambda\alpha}^\dagger (1 + \gamma^5) \bar{e}_\lambda^\dagger) - m_\lambda^2 (\bar{d}_\lambda^\dagger C_{\lambda\alpha}^\dagger (1 - \\
 & \gamma^5) \bar{e}_\lambda^\dagger)] - \frac{8}{3} \frac{m_\lambda^2}{M^2} H (\bar{e}_\lambda^\dagger \bar{e}_\lambda^\dagger) - \frac{8}{3} \frac{m_\lambda^2}{M^2} H (\bar{d}_\lambda^\dagger \bar{d}_\lambda^\dagger) + \frac{5\alpha_w}{2\sqrt{2}} \frac{m_\lambda^2}{M^2} \phi^0 (\bar{e}_\lambda^\dagger \gamma^5 \bar{e}_\lambda^\dagger) - \\
 & \frac{5\alpha_w}{2\sqrt{2}} \frac{m_\lambda^2}{M^2} \phi^0 (\bar{d}_\lambda^\dagger \gamma^5 \bar{d}_\lambda^\dagger) + \bar{X}^\dagger (\partial^\mu - M^2) X^\dagger + \bar{X} (\partial^\mu - M^2) X + \bar{X}^\dagger (\partial^\mu - \\
 & \frac{M^2}{\alpha_h}) X^\dagger + \bar{Y} \partial^\mu Y + i g_{\nu\omega} W_\mu^\alpha (\partial_\nu \bar{X}^\dagger X^\dagger - \partial_\nu \bar{X}^\dagger X^\dagger) + i g_{\nu\omega} W_\mu^\alpha (\partial_\nu \bar{X}^\dagger X^\dagger - \\
 & \partial_\nu \bar{X}^\dagger X^\dagger) + i g_{\nu\omega} W_\mu^\alpha (\partial_\nu \bar{X}^\dagger X^\dagger - \partial_\nu \bar{X}^\dagger X^\dagger) + i g_{\nu\omega} W_\mu^\alpha (\partial_\nu \bar{X}^\dagger X^\dagger - \\
 & \partial_\nu \bar{X}^\dagger X^\dagger) + i g_{\nu\omega} Z_\mu^0 (\partial_\nu \bar{X}^\dagger X^\dagger - \partial_\nu \bar{X}^\dagger X^\dagger) + i g_{\nu\omega} A_\mu (\partial_\nu \bar{X}^\dagger X^\dagger - \\
 & \partial_\nu \bar{X}^\dagger X^\dagger) - \frac{1}{2}g M [\bar{X}^\dagger X^\dagger H + \bar{X}^\dagger X^\dagger H + \frac{1}{\alpha_h} \bar{X}^\dagger X^\dagger H] + \\
 & \frac{1}{2\alpha_h} i g M [\bar{X}^\dagger X^\dagger \phi^\dagger - \bar{X}^\dagger X^\dagger \phi] + \frac{1}{2\alpha_h} i g M [\bar{X}^\dagger X^\dagger \phi^\dagger - \bar{X}^\dagger X^\dagger \phi] + \\
 & i g M_{\nu\omega} [\bar{X}^\dagger X^\dagger \phi^\dagger - \bar{X}^\dagger X^\dagger \phi] + \frac{1}{2}i g M [\bar{X}^\dagger X^\dagger \phi^0 - \bar{X}^\dagger X^\dagger \phi^0]
 \end{aligned}$$

There are many other quantum field theories.

The quantum Hall effect can for example be described by a “Chern-Simons” quantum field theory.

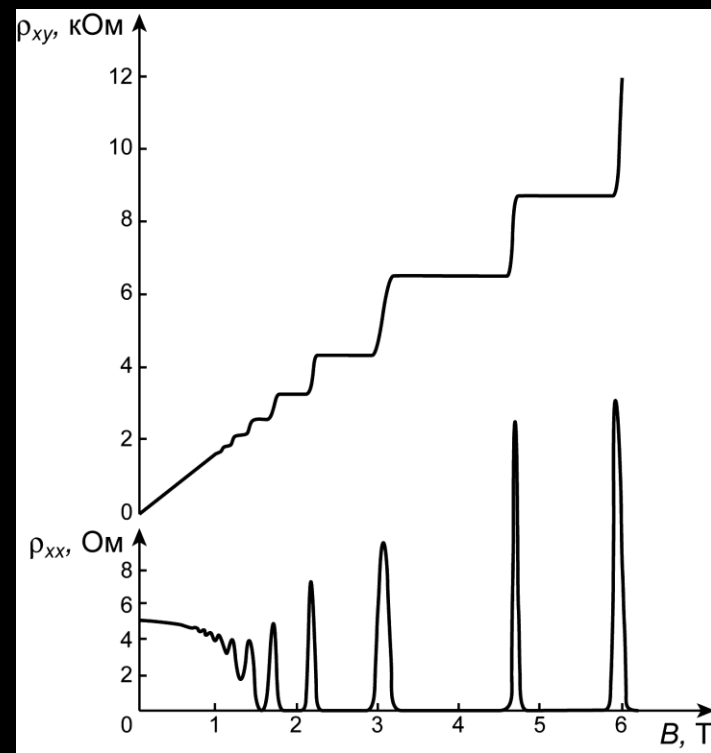
Quantum field theory is a general theoretical framework.

$$S = \frac{k}{4\pi} \int (A \wedge dA + A \wedge A \wedge A)$$



1. Electron Current
2. Hall Element/Sensor
3. Magnets
4. Magnetic Field
5. Power Source

- A. Basic Diagram
- B. Reversing the current
- C. Reversing the magnetic field
- D. Reversing both





# THE NEXT STEP

Classical mechanics  
(no gravity)  
(infinite speed of light)



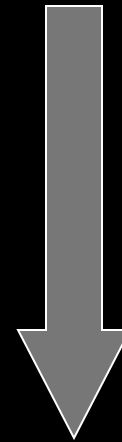
Newtonian gravity  
(infinite speed of light)



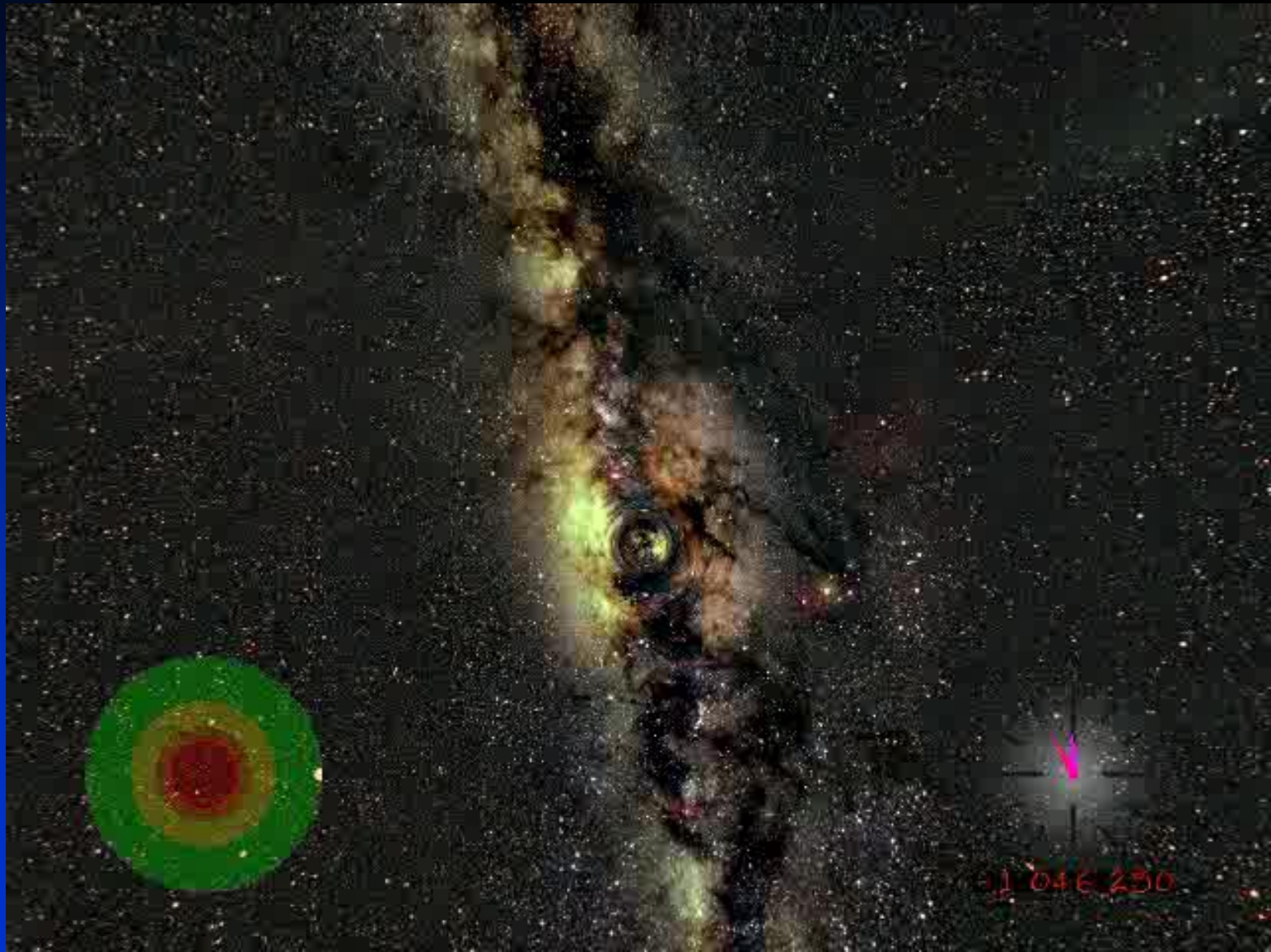
Special relativity  
(no gravity)  
(finite speed of light)

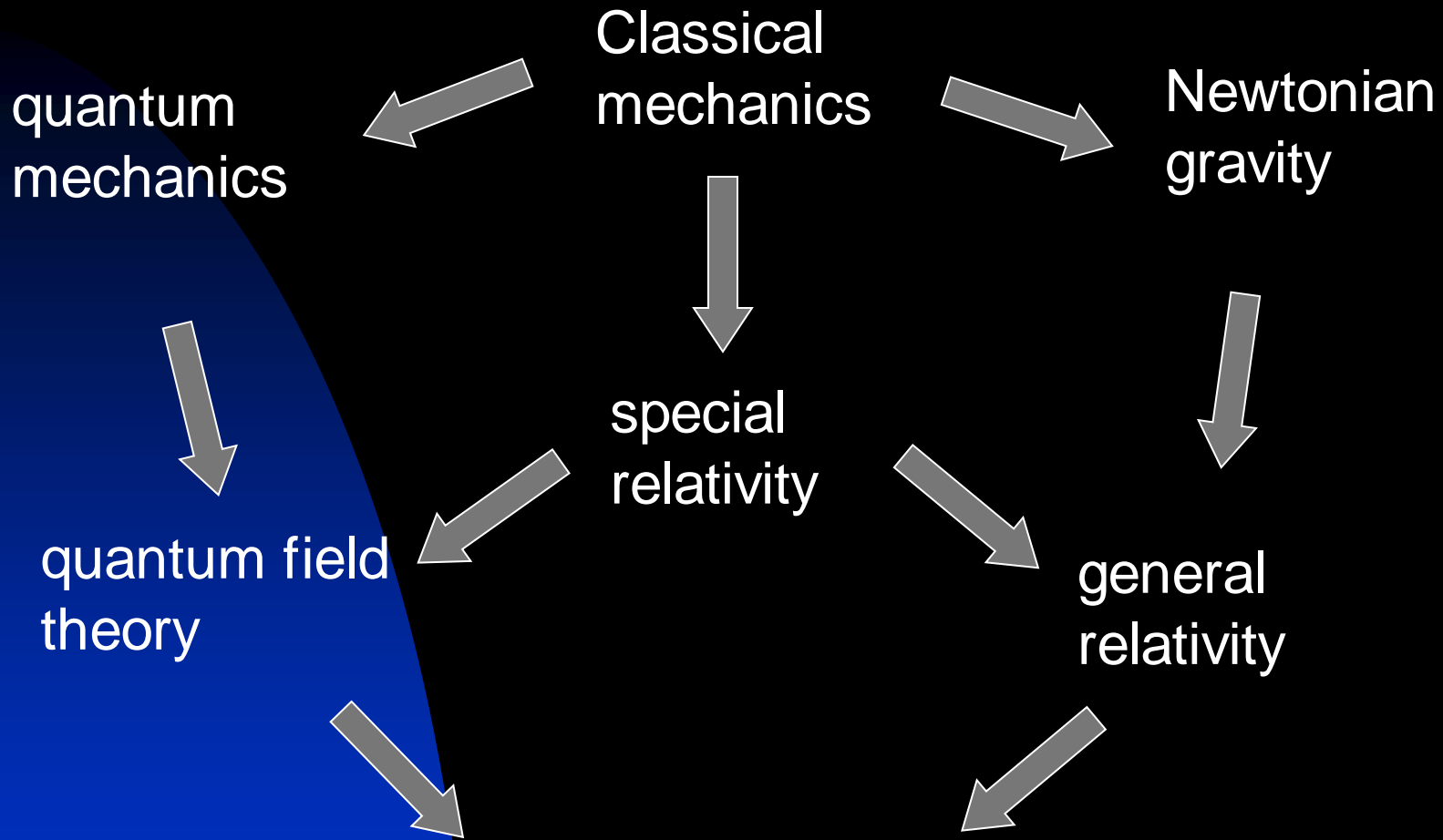


**General relativity**  
(gravity)  
(finite speed of light)



**General relativity:** time and space are curved. In extreme situations black holes can come into existence.





## Quantum gravity

(finite speed of light, wave=particle, gravity)

Can we not simply add quantum field theory and classical general relativity?

**No:** Heisenberg's uncertainty principle indicates that there should be a fundamental uncertainty in the gravitational field as well

**No:** Unitarity of quantum mechanics would be destroyed. (More later).

Can we not treat general relativity as if it were a quantum field theory?

**No:** general relativity is non-renormalizable, would get a theory with infinitely free parameters.

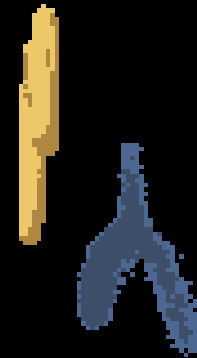
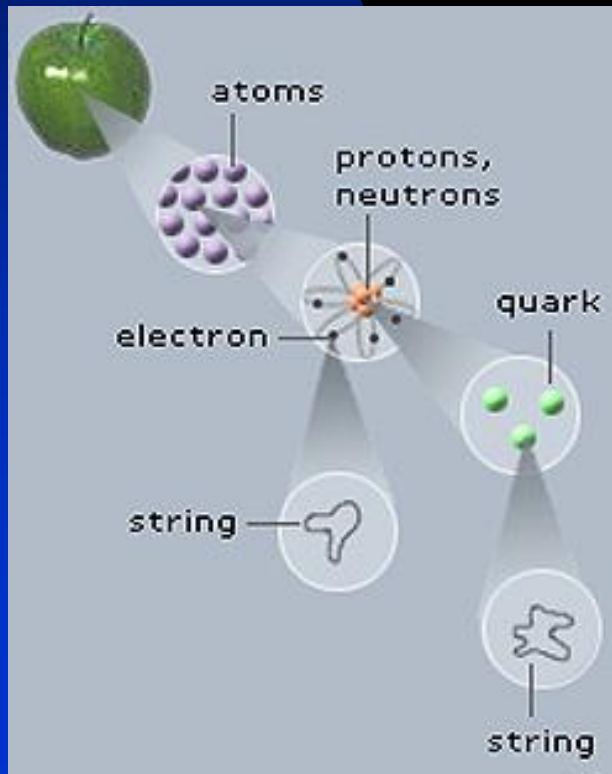
# Who cares about quantum gravity anyway?

Well, I do. I just want to know what the world is like.

Crucial for the understanding of the big bang, the singularity inside black holes, and could give rise to various interesting experimental signatures in colliders (somewhat unlikely) and the universe (not quite as unlikely).

**String theory** is right now the only consistent **theoretical framework** which contains a quantum theory of gravity.

The basic postulate of string theory is that all particles are really different vibrational modes of a very small string.



What does string theory tell us about black holes?

## Black Holes in Classical Gravity:

Completely characterized by their mass and possibly a few other quantum numbers (no hair theorem).

Equilibrium Thermodynamics	Black Hole Mechanics Carter, Bardeen, Hawking
0th law $T = \text{const.}$	0th law $\kappa = \text{const.}$
1st law $dE = T dS$	1st law $dM = \kappa / (8\pi G) dA$
2nd law $dS \geq 0$	2nd law $dA \geq 0$

## Black hole in semiclassical gravity

Hawking showed that if you couple quantum fields to a classical black hole, it produces perfect black body radiation with temperature

$$T = \frac{\hbar c^3}{8\pi G M}$$

This implies that the entropy should be identified with

$$S = \frac{A}{4\ell_P^2}$$

Black holes evaporate! A 10000kg black hole evaporates in 1 second.



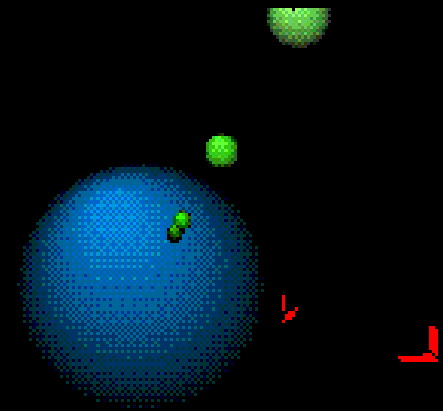
# Quantum field theory + classical gravity is not unitary:

If you make a black hole, after it settles down, the geometry outside the black hole is determined by the mass of the black hole only. (no hair theorem).

The radiation carries no information about the interior of the black hole, unless we give up causality/locality.

Thus the final state depends only on the mass of black hole, but not on what went into it: information loss paradox (pure states evolve into mixed states)

Hawking: give up quantum mechanics??



Nevertheless, the work of Hawking strongly suggests that we should take the thermodynamic analogy seriously.

If so, a black hole represents  $\sim e^S$  microscopic degrees of freedom. What are these degrees of freedom? Are these the fundamental degrees of freedom of quantum gravity?

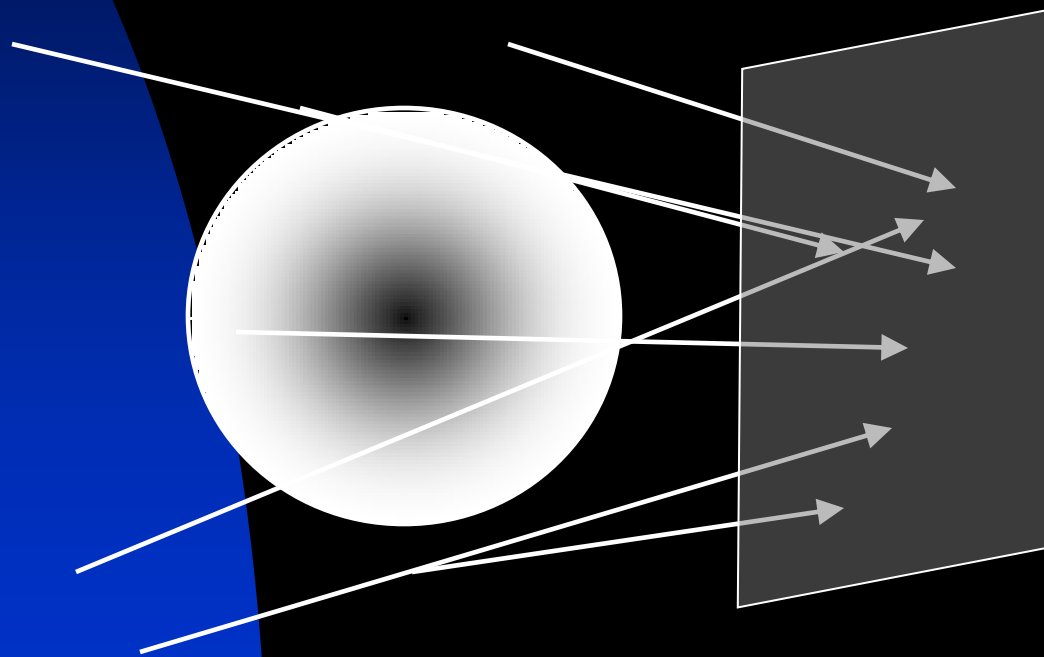
There is something very strange about these degrees of freedom:

Normal local degrees of freedom:  $S \sim \text{volume}$

Quantum gravity degrees of freedom:  **$S \sim \text{area}!!$**

Quantum gravitational degrees of freedom behave like ordinary degrees of freedom in one dimension less.

## HOLOGRAPHY



Holographic screen

The fundamental degrees of freedom of quantum gravity are highly **non-local** and represent some sort of quantum geometries.

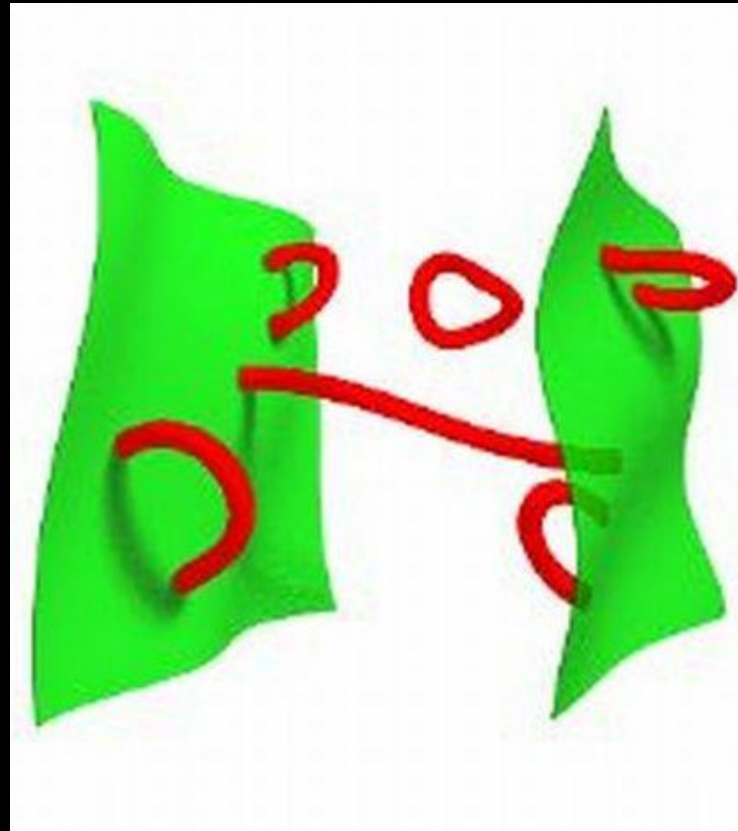
The standard local picture of gravity emerges from these non-local degrees of freedom only after we do a suitable averaging over them (similar to what one does in thermodynamics).

In this sense, spacetime and gravity are **emergent phenomena**.

# What about black holes in string theory?

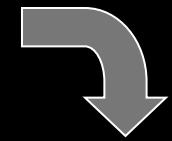
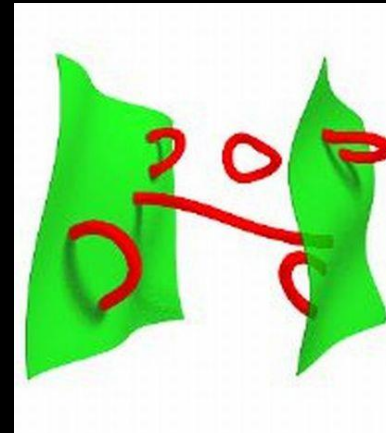
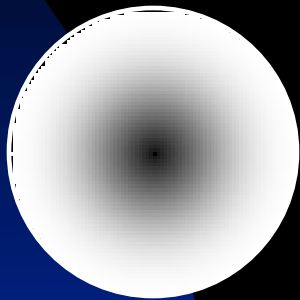
In order to make a very heavy object, use “D-branes”.

D-branes are dynamical objects on which strings can end. At low energies, they are described by a conventional quantum field theory.



Strominger and Vafa managed in 1997 to do the following:

black  
hole



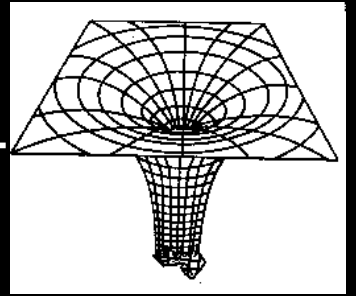
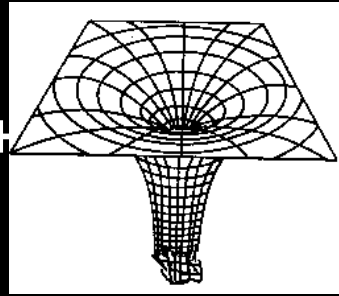
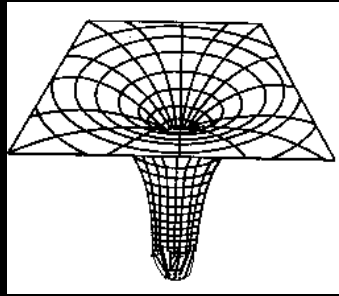
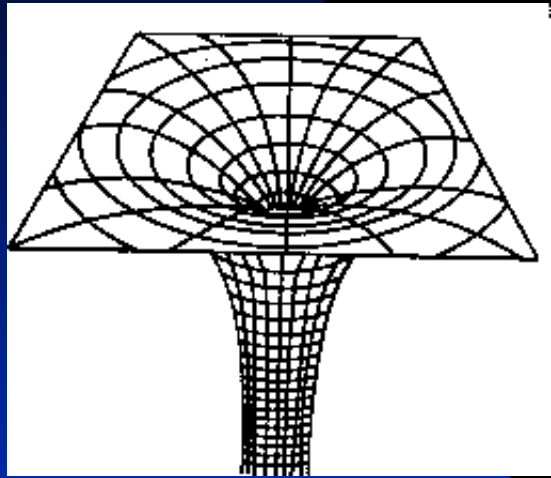
Reproduce  
entropy

Unfortunately, this does not work for generic black holes, and it is not possible to follow the individual degrees of freedom.

Later work, pioneered by Mathur, has allowed us to follow the individual degrees of freedom in certain cases and indeed:

- Generic states do not correspond to classical geometries.
- Certain non-generic states do correspond to classical geometries, but these geometries are very wobbly.
- Classical smooth geometries arise after an analogue of the thermodynamic limit.

For a review, see e.g. Balasubramanian, JdB, El-Showk, Messamah, *Class. Quant. Grav.* 25 (2008)



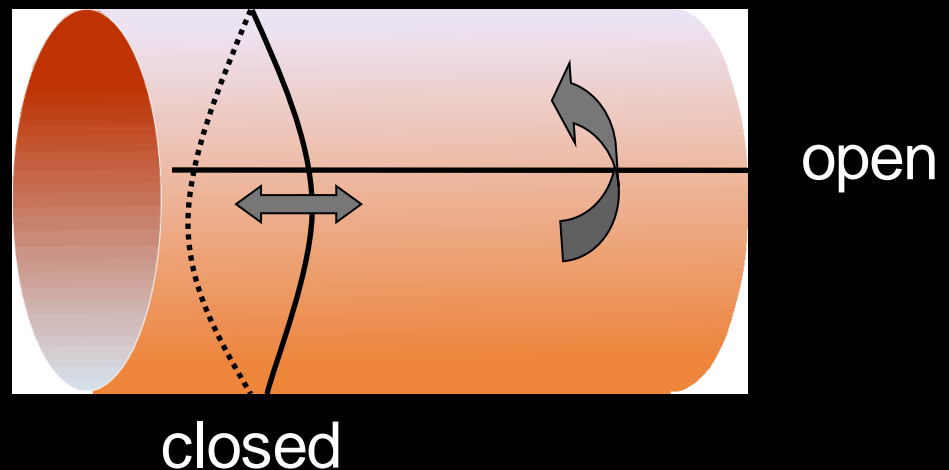


In 1997, a more powerful quantitative setup was found in which the degrees of freedom of quantum gravity are understood and which is manifestly holographic:

## The AdS/CFT correspondence (Maldacena)

Open strings  $\leftrightarrow$  D-branes  $\leftrightarrow$  Quantum Field Theory  
Closed strings  $\leftrightarrow$  Gravity

Basic idea: open-closed string duality



String theory in Anti-de Sitter space is equivalent to a quantum field theory on the boundary

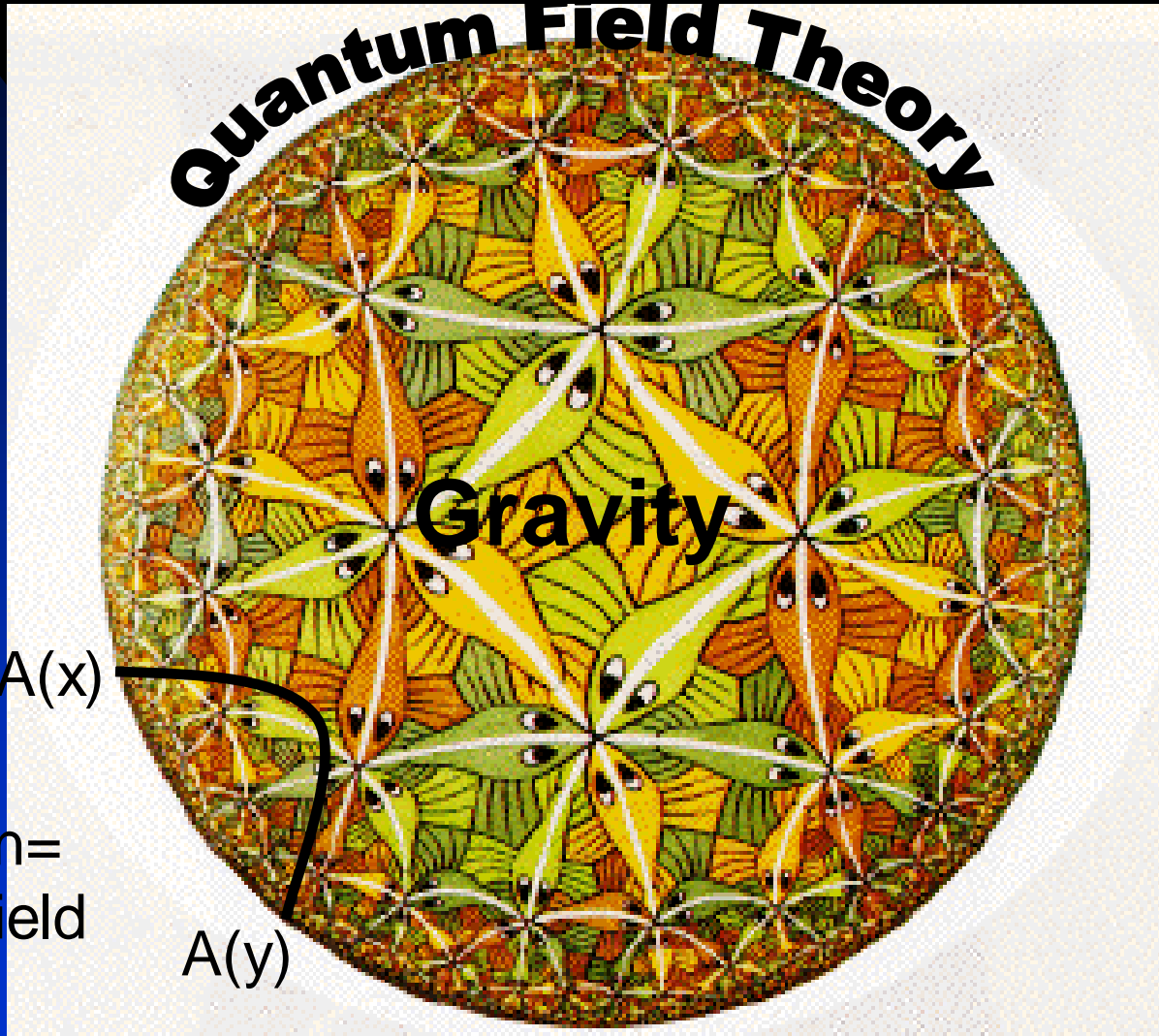
**Quantum Field Theory**

**Gravity**

$A(x)$

$A(y)$

Extra  
dimension=  
scale in field  
theory



## Features:

- Holography is manifest.
- Boundary space and time are well-defined.
- Space and time in the interior are not.
- Boundary theory is a conformal field theory: a field theory without a length scale (as in phase transitions)

$$\langle A(x)A(y) \rangle \gg |x - y|^{-2\phi}$$

$$\langle A(x)A(y) \rangle \gg e^{-|x - y|/L}$$

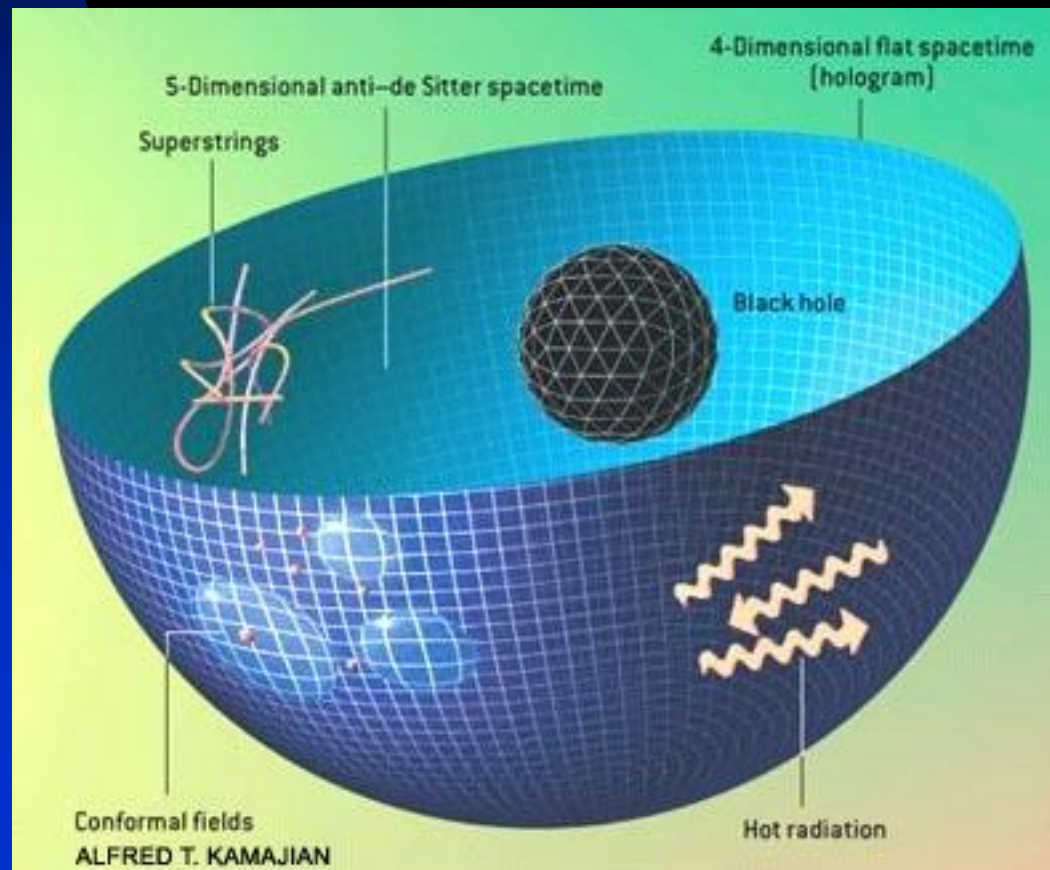
and not

- The Anti-de Sitter space is weakly curved if the field theory on the boundary is strongly coupled, and vice versa.

## Back to black holes:

A black hole in anti-de Sitter space corresponds to field theory at a finite temperature, i.e. a thermal state.

Since the field theory is unitary, black hole creation and evaporation **must be a unitary process**.



But hey..... we don't live in anti-de Sitter space, what are we talking about here?

→ Well, just like there are all kinds of quantum field theories, there are all kinds of string theories. The way gravity appears in all these string theories is universal. Therefore, lessons learned about quantum gravity are believed to be to a large extent model independent.

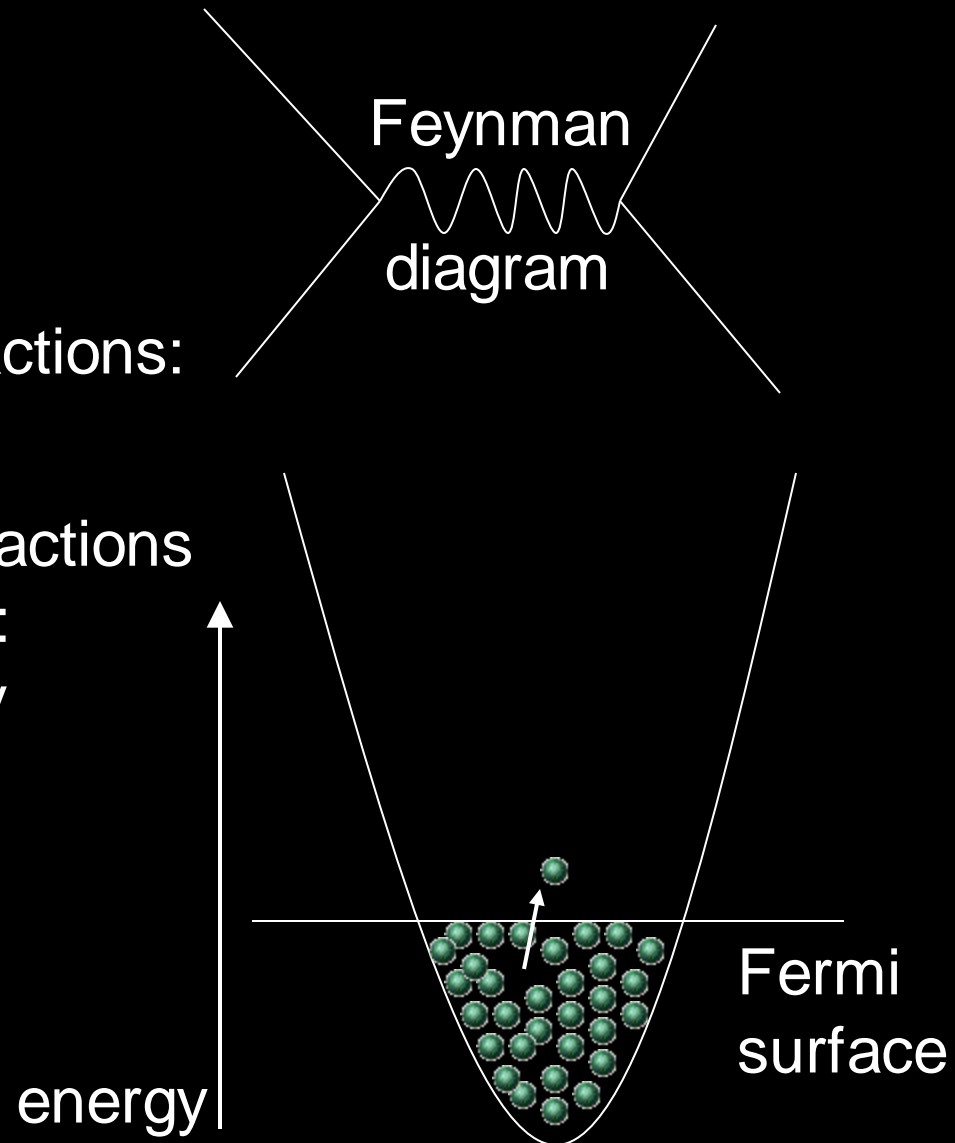
The information loss problem is now solved in principle. What is wrong with Hawking's original argument: **non-locality**. Hawking agrees since 2004 that QM is OK. Many open questions remain.

# Applications of black holes in Anti-de Sitter space-times.

## Easy problems

Few particles, weak interactions:  
perturbation theory

Many particles, weak interactions  
Fermi liquids, solids:  
effective field theory



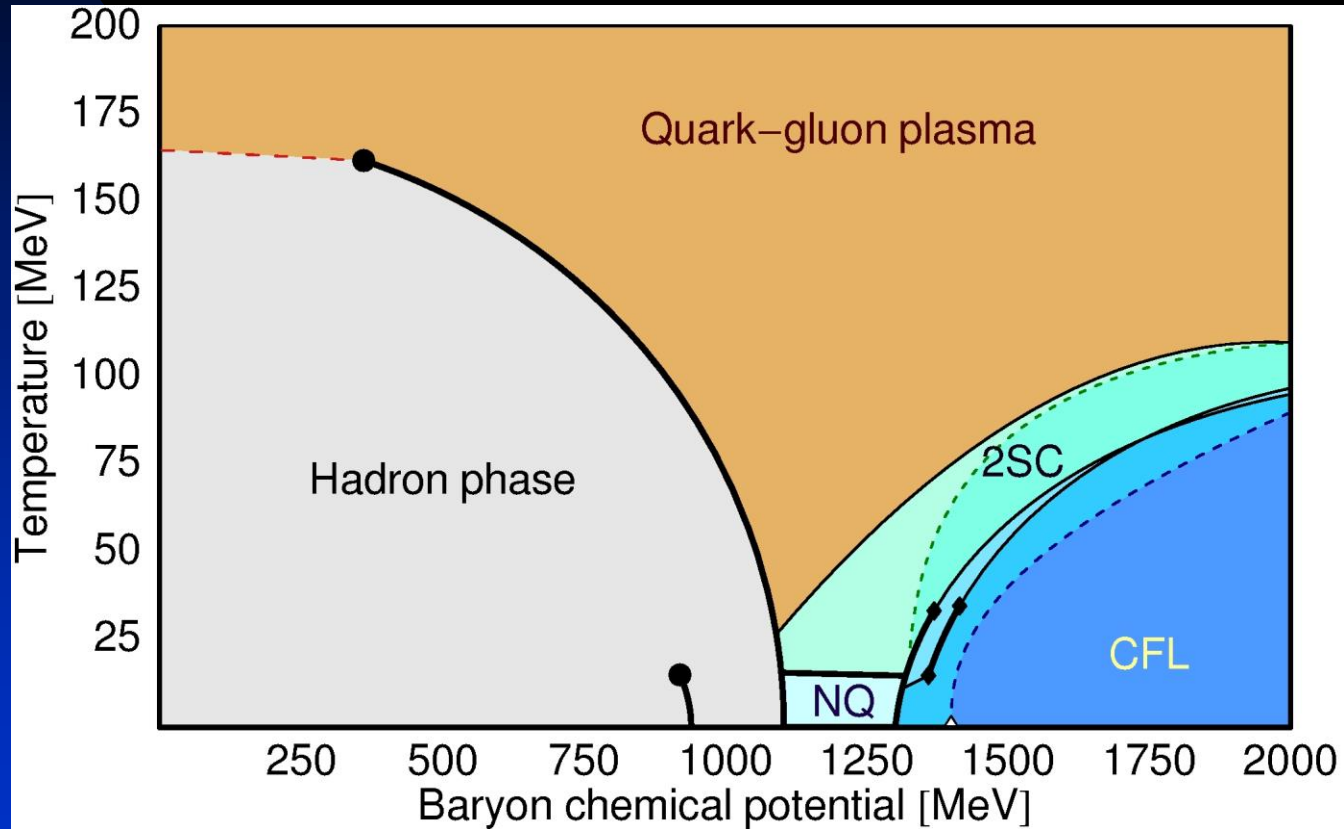
## Difficult problems:

Strongly coupled fluids like the quark-gluon plasma

Strongly coupled electron systems such as those appearing in high- $T_c$  superconductors

All standard approaches to these problems have been quite unsuccessful.

# Quark-Gluon Plasma



hep-ph/0503184



Smash gold atoms on top of each other  
(Brookhaven)



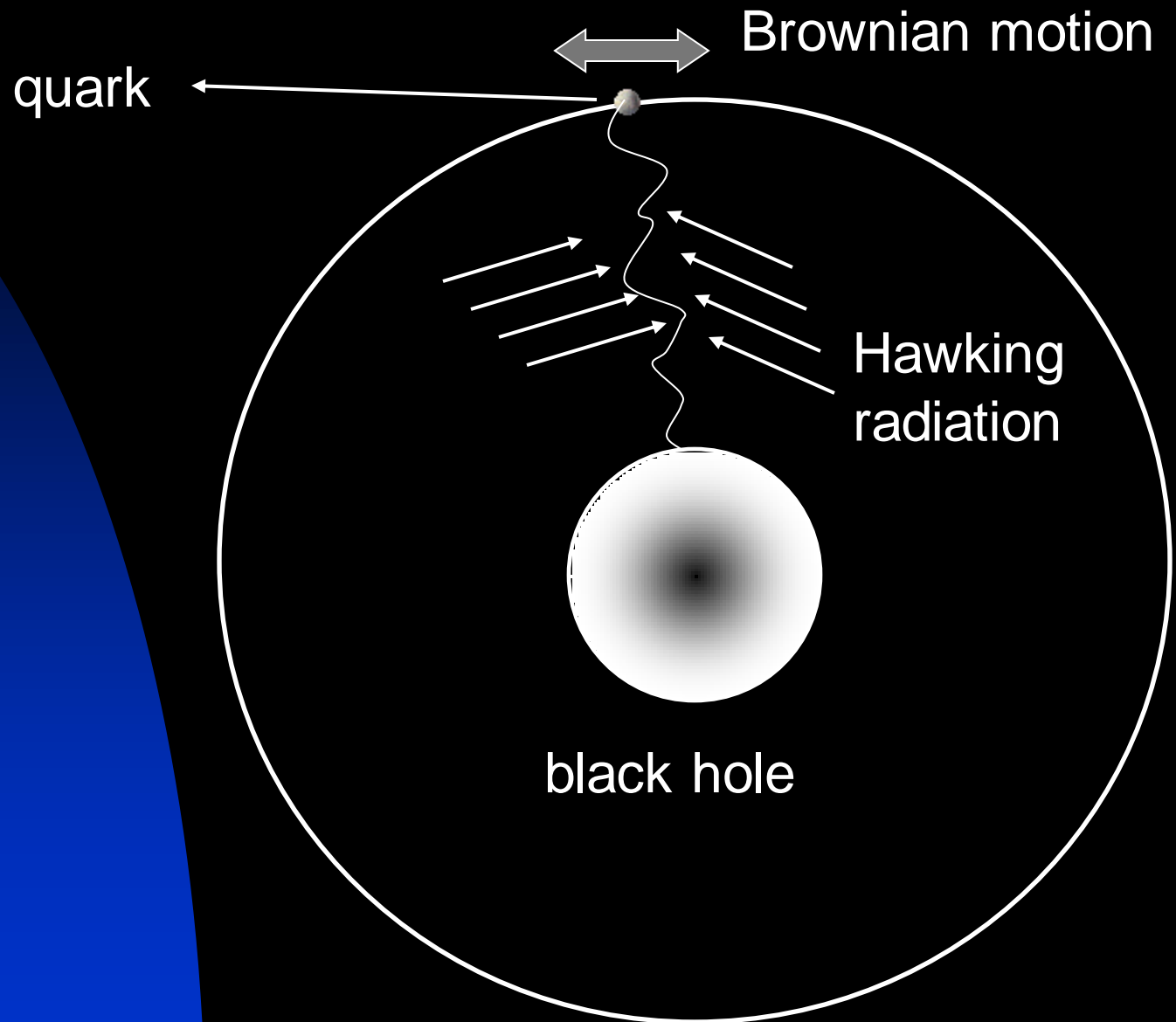
If one models the quark-gluon plasma by a perfect fluid, one gets good agreement with the data.

Big surprise: the viscosity of this liquid is the smallest ever observed.

At high energies, QCD can be approximated by a strongly coupled scale invariant field theory: use the **AdS/CFT correspondence**.

From the graviton propagator in a black hole background in AdS one obtains:  $\frac{1}{s} \sim \frac{1}{4^{1/4}}$  agrees with experiment up to  $O(1)$ .

(Kovtun, Son, Starinets)

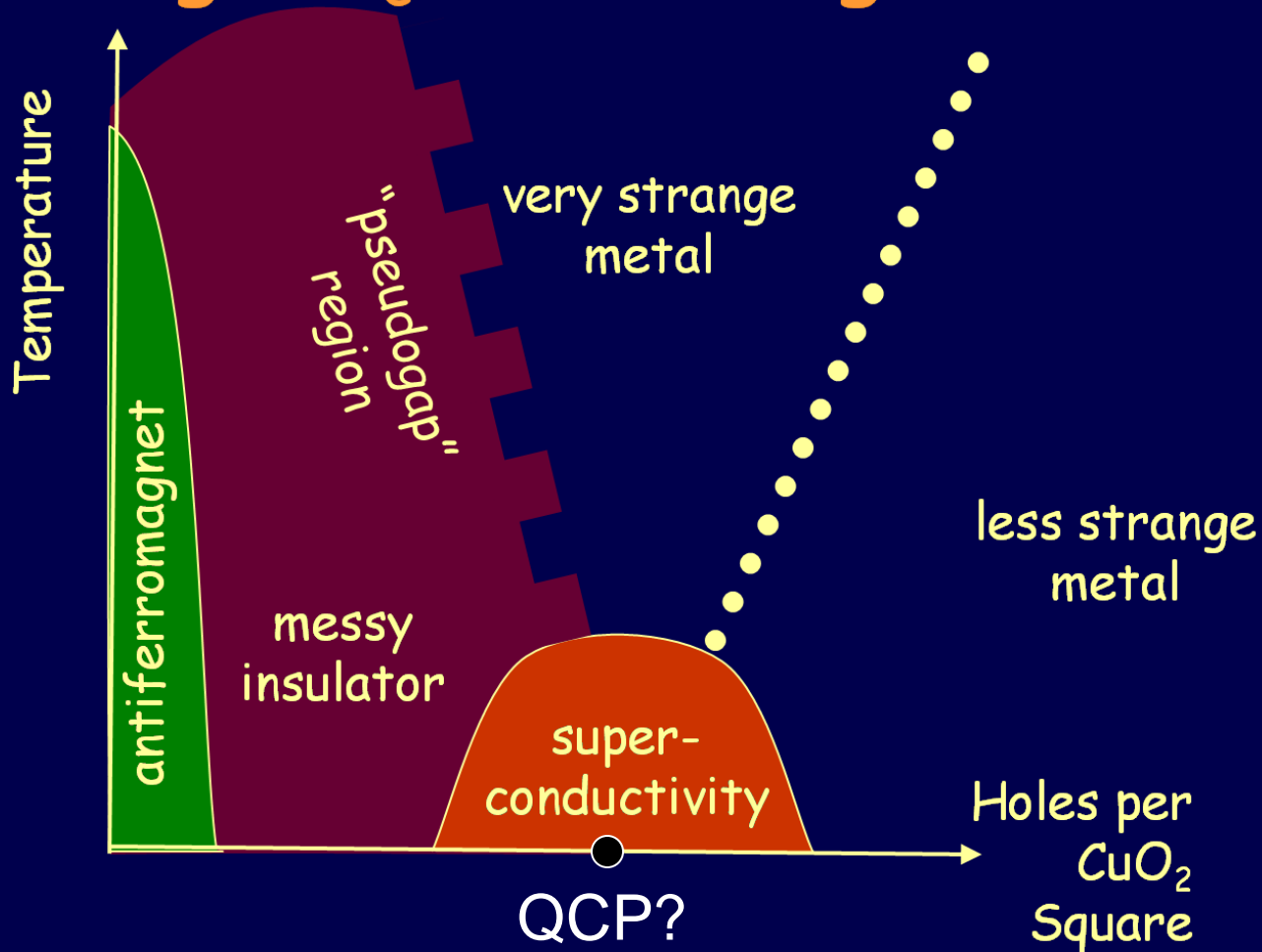


JdB, Hubeny, Rangamani, Shigemori, arxiv:0812.5112

Atmaja, JdB, Shigemori, arxiv: 1002:2429

# High-T<sub>c</sub> superconductors

## High-T<sub>c</sub> Phase Diagram

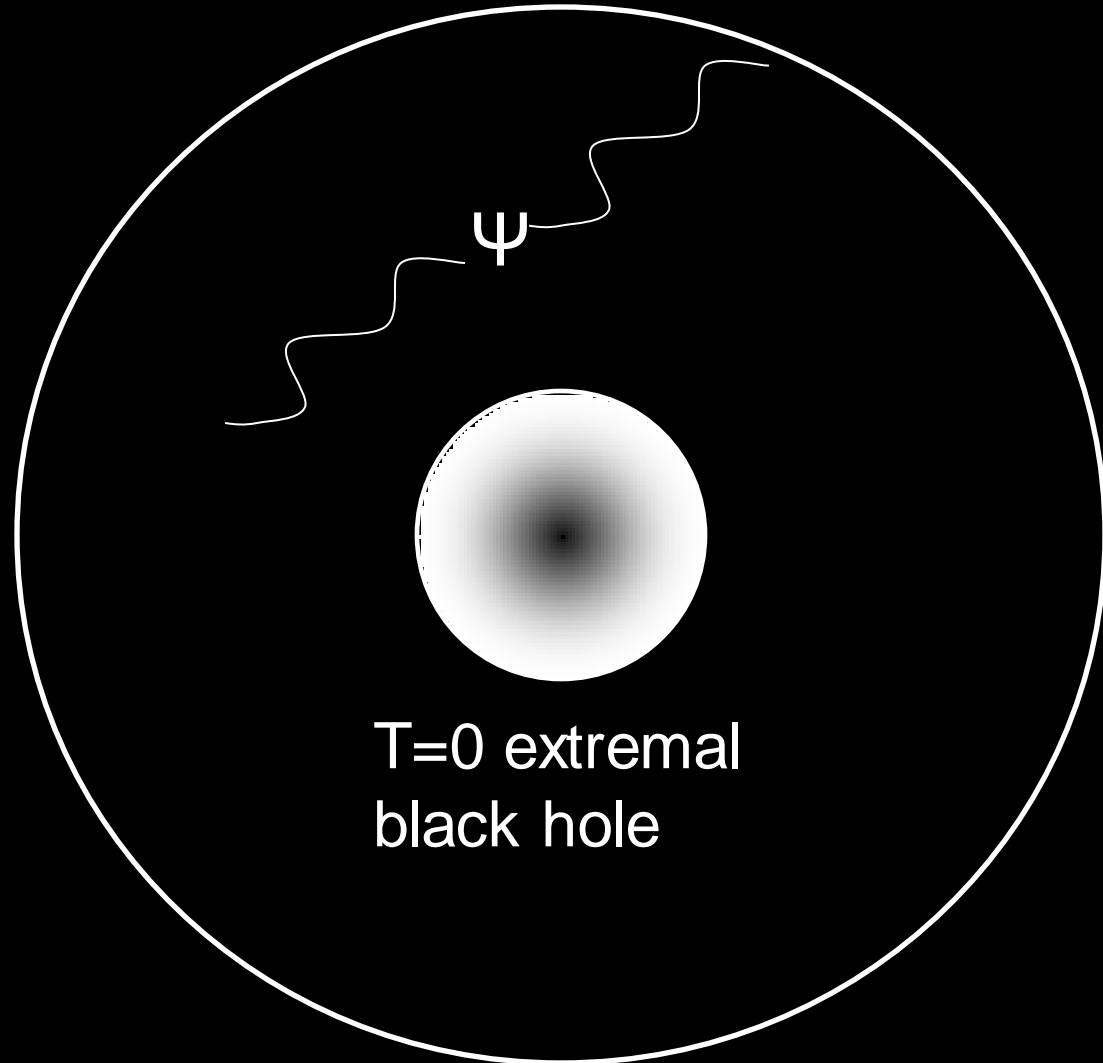


Strange metal phase is believed to be driven by a **quantum critical point** (QCP).

Quantum critical points correspond to phase transitions at zero temperature. They are driven by quantum rather than thermal fluctuations.

QCP may well have large entropy. To “save” third law an instability has to appear: superconductivity. QCP sits under the superconducting dome.

Quantum Critical Point: strongly coupled scale invariant system. Try to model using the AdS/CFT correspondence.



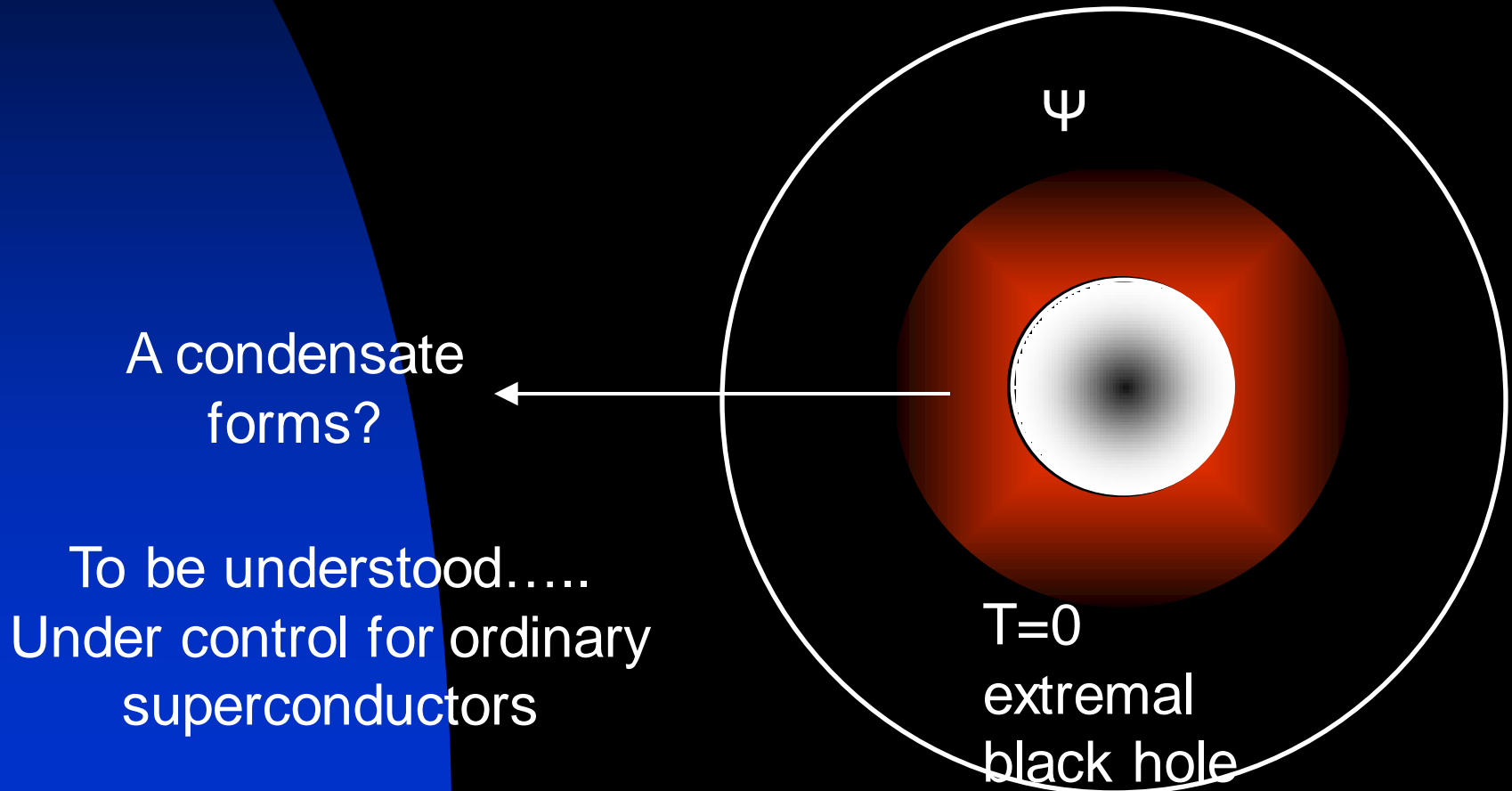
Lee

Cubrovic, Zaanen, Schalm

Faulkner, Liu, McGreevy, Vegh

Can reproduce various aspects of the strange metal phase, including the linear resistivity. Not a robust feature though.

What about the superconducting instability?



# Summary

- String theory is a versatile theoretical framework, just like quantum field theory, which can be applied to many different situations. We do not yet have the analogue of the standard model for string theory.
- We have learned many things about the nature of quantum gravity and its non-local features. For example, we have shown that large smooth geometries may nevertheless be highly non-classical. JdB, El-Showk, Messamah, van den Bleeken
- We have in principle resolved the information paradox.
- String theory can provide a (non-precise) phenomenological description of various strongly coupled systems.



## OPEN PROBLEMS

- A better understanding of quantum gravity in time-dependent space-times like our universe. Time evolution=renormalization group flow?
- Find more experimental evidence for string theory.
- Understand in more detail how information is encoded in Hawking radiation.
- Understand generic black holes in string theory.
- Explain the physics local observers see. Local information is encoded in a very complicated way in the AdS/CFT correspondence. What happens to an observer who falls into a black hole?

Best guess:

