GEOMETRY, LIGHT, AND A WEE BIT OF MAGIC

Ulf Leonhardt University of St Andrews









St Andrews















Mirage



Fermat's Principle - the principle of the shortest optical path



Invisibility cloak?



Stealth technology



Optical camoflage



The secret of optical camoflage



Invisibility: Invisible Man versus Invisible Woman





transparency

Fermat's Principle - the principle of the shortest optical path



Gravitational lens







Conformal maps

Colmographia, siue Descriptio vniuersi Orbis, Petri Apiani & Gemmæ Frisi, Ma= thematicorum insignium_, iam demum inte= gritati suæ restituta_.

Adicăi funt alij, tum Gemmz Frifij, tum aliorum Außorum eius argumentă Tradatus at Libelli varij, quorum feriem verfa pagina demonficat.



Antuerpiz, ex Officina Ioannis VVithagij.





Mercator projection





[Leonhardt, Science **312**, 1777 (2006)]

Virtual space



Physical space





[Pendry, Schurig and Smith, Science **312**, 1780 (2006)]

Virtual space



Physical space





Maxwell's electromagnetism and Einstein's general relativity

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \nabla \cdot \vec{B} = 0, \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{f}, \nabla \cdot \vec{D} = g$$

The covariant free-space Maxwell equations are equivalent to electromagnetism in a material medium (Tamm, 1924; Plebanski, 1960).

$$\vec{D} = \varepsilon \varepsilon \vec{E} + \frac{\vec{W}}{c} \times \vec{H}, \quad \vec{B} = \frac{\mu}{\varepsilon_{o}c^{2}} \vec{H} - \frac{\vec{W}}{c} \times \vec{E}$$
$$\vec{E} = \mu^{ij} = \mp \frac{\sqrt{-g}}{g_{00}} g^{ij}, \quad \vec{W}_{i} = \frac{g_{0i}}{g_{00}}$$





Patent office



Practical demonstrations

[Schurig et al., Science Express, October 19th (2006)]



QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.



Wegener, Linden @ Karlsruhe

Cloaking device for electromagnetic microwaves





cyl.	r	S	μ_r
1	0.260	1.654	0.003
2	0.254	1.677	0.023
3	0.245	1.718	0.052
4	0.230	1.771	0.085
5	0.208	1.825	0.120
6	0.190	1.886	0.154
7	0.173	1.951	0.188
8	0.148	2.027	0.220
9	0.129	2.110	0.250
10	0.116	2.199	0.279

Challenge: Broadband invisibility







Problems: * anomalous dispersion, * infinite speed of light at inner lining [Leonhardt and Philbin, New J. Phys. **8**, 247 (2006)].

Ideas from Non-Euclidean Geometry



The idea







Report

Broadband Invisibility by Non-Euclidean Cloaking

Ulf Leonhardt $^{1,2} \ast$ and Tomás $Tyc^{2,3}$



Ruby glass



Lycurgus Cup Roman, 4th century AD Ruby Glass of the 20th Century Book 2



Particles absorb at different wavelengts depending on the size of particles



The resolution limit of imaging, established around 1870















Negative Refraction Makes a Perfect Lens

J.B. Pendry

Condensed Matter Theory Group, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom (Received 25 April 2000)

With a conventional lens sharpness of the image is always limited by the wavelength of light. An unconventional alternative to a lens, a slab of negative refractive index material, has the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner. Such "superlenses" can be realized in the microwave band with current technology. Our simulations show that a version of the lens operating at the frequency of visible light can be realized in the form of a thin slab of silver. This optical version resolves objects only a few nanometers across.



Negative refraction and perfect lens



Xiang Zhang et al. (a) Berkeley

[Leonhardt and Philbin, New J. Phys. 8, 247 (2006)]

Negatively refracting water





[Aaron Danner, Singapore]

Casimir effect

QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

QuickTime[™] and a TIFF (Uncompressed) decompressor are needed to see this picture.

Capasso

Casimir

Theory: Casimir 1948 Precision experiment: Lamoreaux, PRL 78, 5 (1997)









Quantum levitation



[Leonhardt and Philbin, New J. Phys. 9, 254 (2007)]

Challenge: intuition versus Lifshitz theory



Challenge: theory of nanomachines

$$\begin{aligned}
 \varepsilon_1(\omega) & \mathbf{r} & \mathbf{r} \\
 \mu_1(\omega) & \mathbf{r}' & | \varepsilon_2(\omega) \\
 \mathbf{r}' & | \mu_2(\omega) \\
 x=0 & x=a
 \end{aligned}$$

Static Casimir effect: Lifshitz 1955

Motion: Philbin and Leonhardt, New J. Phys. **11**, 033035 (2009).

No quantum friction between moving plates

QuickTime[™] and a Cinepak decompressor are needed to see this picture.





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Maxwell's fish eye makes a perfect lens Maxwell 1854

Luneburg 1944: Stereographic projection



Perfect imaging without negative refraction

[Leonhardt, New J. Phys. 11, 093040 (2009)]



$$n = \frac{2n_0}{1 + r^2 / r_0^2}$$

Index contrast: factor of 2



Lipson Group, Cornell University

Similar to carpet cloaking [LH. Gabrielli, J. Cardenas, C.B. Poitras, and M. Lipson, Nature Photonics 3, 461 (2009)]

Light in 1D moving media

 $-\int \frac{dz}{V_{\pm}}$ $u \pm c_n$ t_ = ' v_± ucn) ± $t_{\pm} = t' \mp \frac{z'}{c}$

°0°

Singularity

Space-time transformations: moving media

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \nabla \cdot \vec{B} = 0, \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}, \nabla \cdot \vec{D} = g$$

Constitutive equations of electromagnetism in a space-time geometry:

$$\vec{D} = \varepsilon_{0}\varepsilon\vec{E} + \frac{\vec{W}}{c}\times\vec{H}, \quad \vec{B} = \frac{\mu}{\varepsilon_{0}c^{2}}\vec{H} - \frac{\vec{W}}{c}\times\vec{E}$$

$$\vec{\varepsilon}^{ij} = \mu^{ij} = \mp \frac{\sqrt{-g'}}{g_{00}}g^{ij}, \quad \vec{W}_{i} = \frac{g_{0i}}{g_{00}}$$

$$\vec{W}_{i} = \frac{\mu^{2} - 1}{1 - \mu^{2}\mu^{2}/c^{2}} \quad u_{i}$$

Moving media

Artificial event horizons

Piotr Pieranski

QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

Hawking radiation of black holes

they must produce a response in the scintillator comparable with that produced by a conventional charged particle. As mentioned earlier, the EAS studied by Ramana Murthy were almost two orders of magnitude less energetic than those used by us and also a much smaller time range was examined. Since the peak in our distribution occurs after the 18 μ s time interval investigated by Ramana Murthy, the results cannot be directly compared. But a statistical examination of his published data using David's technique indicates that there is less than 5% probability that the data is from a uniform distribution. We note that, subjectively, the observed distribution seems to rise in the period prior to 13 us before the shower arrival. This is not inconsistent with our observations.

It is possible than an explanation may be found for these results without invoking the existence of tachvons, A. G. Gregory has pointed out to us that fission or spallation in the interstellar medium or production of associated particles at the source might account for the correlated arrival of cosmic rays. We note, however, that unless particles are produced with closely similar rigidity and velocity vectors they are unlikely to remain associated for long in the interstellar or source magnetic fields. A closely related problem has been discussed by Weekes⁸ in connection with pulsar periodicities in cosmic-ray arrival times.

We conclude that we have observed non-random events preceding the arrival of an extensive air shower. Being unable to explain this result in a more conventional manner, we suggest that this is the result of a particle travelling with an apparent velocity greater than that of light.

We thank Dr A. G. Gregory and Professors C. A. Hurst and J. R. Prescott for comments, Mr K. W. Morris for advice on statistical analysis, and Ms J. M. Taylor for chart reading. One of us (R.W.C.) holds a Queen Elizabeth Fellowship. The University of Calgary is thanked for the loan of equipment

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M. N., Phys. Rev., 171, 1357 (1968). Yeh, N., Linsker, R., Phys. Rev. D., ch, G. R., Borenstein, S. R., Strand, Chapman, J. W., Lys, J., Phys. Rev. Lett. Nuovo Cim. Ser. 2, 1, 908 m. Part. cosm. Ray Phys., 10, 170 Rossi, B., Phys. Rev., 92, 441 (1953). 34, 299 (1947) hys. Sci., 223, 129 (1971).

Black hole explosions?

QUANTUM gravitational effects are usually ignored in calculations of the formation and evolution of black holes. The justification for this is that the radius of curvature of spacetime outside the event horizon is very large compared to the Planck length $(G\hbar/c^3)^{1/2} \approx 10^{-33}$ cm, the length scale on which quantum fluctuations of the metric are expected to be of order unity. This means that the energy density of particles created by the gravitational field is small compared to the space-time curvature. Even though quantum effects may be small locally, they may still, however, add up to produce a significant effect over the lifetime of the Universe $\approx 10^{17}$ s which is very long compared to the Planck time $\approx 10^{-43}$ s.

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The purpose of this letter is to show that this indeed may be the case: it seems that any black hole will create and emit particles such as neutrinos or photons at just the rate that one would expect if the black hole was a body with a temperature of $(\kappa/2\pi)(\hbar/2k) \approx 10^{-6} (M_{\odot}/M)K$ where κ i the surface gravity of the black hole 1. As a black hole emit this thermal radiation one would expect it to lose mass. This in turn would increase the surface gravity and so increase the rate of emission. The black hole would therefore have finite life of the order of 10^{71} $(M_{\odot}/M)^{-3}$ s. For a black hole of solar mass this is much longer than the age of the Universe. There might, however, be much smaller black holes which were formed by fluctuations in the early Universe². Am such black hole of mass less than 1015 g would have evaporated by now. Near the end of its life the rate of emiss would be very high and about 10³⁰ erg would be released in the last 0.1 s. This is a fairly small explosion by astronomica standards but it is equivalent to about 1 million 1 Mton hydrogen bombs.

To see how this thermal emission arises, consider (for simplicity) a massless Hermitean scalar field ϕ which obey the covariant wave equation ϕ ; ${}_{ab}g^{ab} = 0$ in an asymptotically flat space time containing a star which collapses to produc a black hole. The Heisenberg operator ϕ can be expressed as

 $\phi = \sum \{f_i a_i + \bar{f}_i a_i^+\}$

where the f_i are a complete orthonormal family of complex valued solutions of the wave equation $f_{i;ab}g^{ab} = 0$ which are asymptotically ingoing and positive frequency-they contain only positive frequencies on past null infinity $I^{-3,4,5}$. The position-independent operators a_i and a_i^+ are interpreted as annihilation and creation operators respectively for incoming scalar particles. Thus the initial vacuum state, the state con taining no incoming scalar particles, is defined by $a_1|0_{-}\rangle = 0$ for all i. The operator ϕ can also be expressed in terms of solutions which represent outgoing waves and waves crossing the event horizon:

$$\phi = \sum \{ p_i b_i + \bar{p}_i b_i^{+} + q_i c_i + \bar{q}_i c_i^{+} \}$$

where the p_i are solutions of the wave equation which are zero on the event horizon and are asymptotically outgoing positive frequency waves (positive frequency on future null infinity I^*) and the q_i are solutions which contain no outgoing component (they are zero on I^*). For the present purposes it is not necessary that the q_i are positive frequence on the horizon even if that could be defined. Because field of zero rest mass are completely determined by their value on I^{-} , the p_i and the g_i can be expressed as linear combinanations of the f_i and the $\overline{f_i}$:

$$p_i = \sum_i \{\alpha_{ii}f_i + \beta_{ii}\bar{f}_i\}$$
 and so on

The β_{ij} will not be zero because the time dependence of the metric during the collapse will cause a certain amount o mixing of positive and negative frequencies. Equating the two expressions for ϕ , one finds that the b_i , which are the annihilation operators for outgoing scalar particles, can be expressed as a linear combination of the ingoing annihilation and creation operators a_i and a_i^*

$$b_i = \sum_{i} \{ \bar{\alpha}_{ij} a_j - \bar{\beta}_{ij} a_j \}$$

Thus when there are no incoming particles the expectation value of the number operator $b_i^*b_i$ of the *i*th outgoing state is

$$\langle 0_{-} | b_{i}^{+} b_{i} | 0_{-} \rangle = \sum_{i} |\beta_{ii}|$$

The number of particles created and emitted to infinity in a gravitational collapse can therefore be determined by calcu lating the coefficients β_{ij} . Consider a simple example in which

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e collapse is spherically symmetric. The angular dependence f the solution of the wave equation can then be expressed in erms of the spherical harmonics Y_{im} and the dependence on tarded or advanced time u, v can be taken to have the orm $\omega^{-1/2} exp$ (i ωu) (here the continuum normalisation is sed). Outgoing solutions p_{imw} will now be expressed as an tegral over incoming fields with the same l and m:

$$p_{\omega} = \int \left\{ \alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} \bar{f}_{\omega'} \right\} d\omega'$$

The lm suffixes have been dropped.) To calculate α_{aver} and ω consider a wave which has a positive frequency ω on I^* opagating backwards through spacetime with nothing crossthe event horizon. Part of this wave will be scattered by he curvature of the static Schwarzschild solution outside the stack hole and will end up on I^- with the same frequency ω . This will give a $\delta(\omega - \omega')$ behaviour in $\alpha_{\omega\omega'}$. Another part the wave will propagate backwards into the star, through he origin and out again onto I. These waves will have a erv large blue shift and will reach I- with asymptotic form

 $C\omega^{-1/2} \exp \{-i\omega\kappa^{-1} \log (v_0 - v) + i\omega v\}$ for $v < v_0$

ad zero for $v > v_{a}$, where v_{a} is the last advanced time at which a particle can leave I^- , pass through the origin and cape to I⁺. Taking Fourier transforms, one finds that for arge ω' , $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ have the form:

$$\omega \approx C \exp \left[i(\omega - \omega)v_0\right](\omega'/\omega)^{1/2}$$

$$\cdot \Gamma(1 - i\omega/\kappa) [-i(\omega - \omega')]^{-1 + i\omega/\kappa}$$

 $z_{**} \approx C \exp [i(\omega + \omega)v_0](\omega'/\omega)^{1/2}$

 $\cdot \Gamma(1 - i\omega/\kappa)[-i(\omega + \omega')]^{-1+i\omega/\kappa}.$

The total number of outgoing particles created in the fremency range $\omega \to \omega + d\omega$ is $d\omega \int_0^{\omega} |\beta_{\mu\nu\nu'}|^2 d\omega'$. From the above pression it can be seen that this is infinite. By considering going wave packets which are peaked at a frequency w and at late retarded times one can see that this infinite mber of particles corresponds to a steady rate of emission at late retarded times. One can estimate this rate in the wing way. The part of the wave from I^* which enters he star at late retarded times is almost the same as the art that would have crossed the past event horizon of the chwarzschild solution had it existed. The probability flux a wave packet peaked at ω is roughly proportional to $|||^{\omega_{2'}} \{\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2\} d\omega$ where $\omega_2' \gg \omega_1' \gg 0$. In the exssions given above for α_{ww} , and β_{ww} , there is a logarithmic ingularity in the factors $[-i(\omega - \omega')]^{-1+i\omega/\kappa}$ and $-i(\omega + \omega')$]^{-1+tw/#}. Value of the expressions on different eets differ by factors of $\exp(2\pi n\omega\kappa^{-1})$. To obtain the correct tio of $\alpha_{\omega\omega'}$ to $\beta_{\omega\omega'}$ one has to continue $[-i(\omega + \omega')]^{-1+i\omega/\kappa}$ in the upper half ω' plane round the singularity and then place ω' by $-\omega'$. This means that, for large ω' ,

$|\alpha_{\mu\mu'}| = \exp(\pi\omega/\kappa) |\beta_{\mu\mu'}|$

From this it follows that the number of particles emitted in his wave packet mode is $(\exp(2\pi\omega/\kappa) - 1)^{-1}$ times the mber of particles that would have been absorbed from a milar wave packet incident on the black hole from I-. But is just the relation between absorption and emission cross tions that one would expect from a body with a temperain geometric units of $\kappa/2\pi$. Similar results hold for ssless fields of any integer spin. For half integer spin one min gets a similar result except that the emission cross stion is $(\exp(2\pi\omega/\kappa) + 1)^{-1}$ times the absorption cross tion as one would expect for thermal emission of fermions. iese results do not seem to depend on the assumption of spherical symmetry which merely simplifies the calcu-

Beckenstein⁶ suggested on thermodynamic grounds that some multiple of κ should be regarded as the temperature of a black hole. He did not, however, suggest that a black hole could emit particles as well as absorb them. For this reason Bardeen, Carter and I considered that the thermodynamical similarity between κ and temperature was only an analogy. The present result seems to indicate; however, that there may be more to it than this. Of course this calculation ignores the back reaction of the particles on the metric, and quantum fluctuations on the metric. These might alter the picture.

Further details of this work will be published elsewhere. The author is very grateful to G. W. Gibbons for discussions and help.

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- Bardeen, J. M., Carter, B., and Hawking, S. W., Commun. math. Phys., 31, 161-170 (1973).
 Hawking, S. W., Mon. Not. R. estr. Soc., 152, 75-78 (1971).
 Penrose, R., in Relativity, Groups and Topology (edit. by de Witt, C. M., and de Witt, B. S.). Les Houches Symmetry
- School, 1963 (Gordon and Breach, New York, 1964). * Hawking, S. W., and Ellis, G. F. R., The Large-Scale Structure of Space-Time (Cambridge University Press, London 1973).
- ⁵ Hawking, S. W., in Black Holes (edit. by de Witt, C. M., and de Witt, B. S), Les Houches Summer School, 1972 (Gordon and Breach, New York, 1973).
- 6 Beckenstein, J. D., Phys. Rev., D7, 2333-2346 (1973).

Absorption and emission by interstellar CH at 9 cm

RYDBECK, Ellder and Irvine¹ have recently detected the 9-cm lines of the ${}^{2}\Pi_{1/2}$, $J = \frac{1}{2}\Lambda$ doublet of interstellar CH. The $F = 1 \rightarrow 1$ transition at 3,335.475 MHz was observed in emission in a wide range of galactic sources ranging from dark clouds to the spiral arms in front of Cassiopea A. The two satellite transitions $F = 0 \rightarrow 1$ (at 3,263.788 MHz) and $F = 1 \rightarrow 0$ (at 3,349.185 MHz) were also observed in emission in several sources.

We have observed the 3.335.475 MHz transition of CH in several southern galactic sources. In RCW38 this line is seen in absorption, while the two satellite lines are seen in emission. In several sources the distribution of CH is found to be extended

The observations were made on December 10 and 11, 1973, with the Parkes 64-m telescope equipped with a 9-cm parametric amplifier having a noise temperature of 150 K. The telescope beam at 9 cm is 6 arc min. The receiver output was analysed by a 512-channel digital correlator producing a spectral resolution of 19.5 kHz.

The Onsala observations¹ of CH emission at 3.335.475 MHz from Cloud 2 and W12 were confirmed. In Cloud 2 we measured an antenna temperature similar to that found with the Onsala 25-m telescope. For W12 it was 0.23 K, about 50% greater than the Onsala value of 0.15 K; however, at a position 5 arc min south (where the continuum intensity had fallen to one seventh of its peak value) the line signal had decreased by only 30%. A similar situation occurred in RCW36. Thus the CH distribution is considerably more extended than the continuum for these HII regions; in Cloud 2 it must be comparable with the 16 arc min beam of the Onsala 25-m telescope.

Hawking radiation: connections

$$k_{\rm B}T = \frac{t_{\rm A}}{2r}$$
, $\alpha = \frac{c^3}{4GM} = \frac{c}{2R}$

M: MASS, R: SCHWARZSCHILD RADIUS

Artificial event horizons in superfluid Helium-3

Kylmälaboratorio Lågtemperaturlaboratoriet

Low Temperature Laboratory

INTERNATIONAL SERIES OF MONOGRAPHS ON PHYSICS + 117

The Universe in a Helium Droplet

GRIGORY E. VOLOVIK

OXFORD SCIENCE RUBLICATIONS

Artificial event horizons in optical fibers (St Andrews project)

Light in 1D moving media

 $-\int \frac{dz}{V_{\pm}}$ $u \pm c_n$ t_ = ' v_± ucn) ± $t_{\pm} = t' \mp \frac{z'}{c}$

°0°

Singularity

Few-cycle pulses in microstructured fibres

QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

Experiment

Probe laser

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General relativity in electrical engineering

[Leonhardt and Philbin, New J. Phys. 8, 247 (2006)]

Einwell and Maxstein

QuickTime[™] and a decompressor are needed to see this picture.

Ulf Leonhardt and Thomas Philbin

GEOMETRY AND LIGHT The Science of INVISIBILITY

