Four historical epochs and four fundamental constants of modern cosmology

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS

The Oskar Klein Memorial Lecture
AlbaNova, Stockholm, 02.12.2010

History of the Universe

Present matter content of the Universe

Dark matter and dark energy

4 fundamental cosmological constants

New fundamental constants expected

Viable one-parameter inflationary models

Homage to Oskar Klein

Conclusions

4 epochs of the Universe evolution

The history of the Universe in one line according to the present paradigm:

$$? \longrightarrow DS \Longrightarrow FRWRD \Longrightarrow FRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \, .$$

Present matter content of the Universe

In terms of the critical density
$\rho_{crit} = \frac{3H_0^2}{8\pi G} \approx 0.9 \times 10^{-29}\ \mathrm{g/cm^3}, \quad \Omega_i = \frac{\rho_i}{\rho_{crit}}, \quad \sum_i \Omega_i = 1$
(neglecting spatial curvature - less than 1%):

- Baryons (p,n) and leptons (e^-) 4.5%
  No primordial antimatter.
- Photons (γ) 4×10^{-5}
  $T_\gamma = (2.72548 \pm 0.00057)\mathrm{K}$ (arXiv:0911.1955)
- Neutrinos ($\nu_e,\ \nu_\mu,\ \nu_\tau$) $< 1\%$

$$\sum_i m_{\nu i} < 0.6\ \mathrm{eV}, \quad \sum_i m_{\nu i} = 94\Omega_\nu h^2\ \mathrm{eV}.$$

- Non-relativistic non-baryonic dark matter 23%
- Dark energy 73%

Dark matter

Dark matter and dark energy are seen through gravitational interaction only – we know the structure of their effective energy-momentum tensor.

DM - non-relativistic, gravitationally clustered.
DE - relativistic, unclustered.
Definition of their effective EMT – through equations (conventional).
DM - through the generalized Poisson equation:

$$\frac{\triangle \Phi}{a^2} = 4\pi G(\rho - \rho_0(t)).$$

$\Phi(\mathbf{r}, t)$ is measured using the motion of 'test particles' in it.
a) Stars in galaxies $\rightarrow$ rotation curves.
b) Galaxies $\rightarrow$ peculiar velocities.
c) Hot gas in galaxies $\rightarrow$ X-ray profiles.
d) Photons $\rightarrow$ gravitational lensing (strong and weak).

Observations: DM is non-relativistic, has a dust-like EMT – $p \ll \epsilon = \rho c^2, \quad p > 0$, collisionless in the first approximation – $\sigma/m < 1\ \mathrm{cm}^2/\mathrm{g}$, and has the same spatial distribution as visible matter for scales exceeding a few Mpc.

Ground experiments: very weakly interacting with baryonic matter, $\sigma < 10^{-43}\ \mathrm{cm}^2$ for $m \sim (50 - 100)$ GeV (there still exist an interesting window with $m \sim 7$ GeV and $\sigma \sim 10^{-40}\ \mathrm{cm}^2$ barely consistent with the results of DAMA, CDMS and CoGeNT, see however the latest paper of the CDMS collaboration arXiv:1011.2482).

Dark energy

Two cases where DE shows itself:
1) inflation in the early Universe – primordial DE,
2) present accelerated expansion of the Universe – present DE.

Quantitative and internally self-consistent definition of its effective EMT - through gravitational field equations conventionally written in the Einstein form:

$$\frac{1}{8\pi G}\left(R_{\mu}^{\nu} - \frac{1}{2}\,\delta_{\mu}^{\nu} R\right) = -\left(T_{\mu\,(vis)}^{\nu} + T_{\mu\,(DM)}^{\nu} + T_{\mu\,(DE)}^{\nu}\right) \;,$$

$G = G_0 = const$ - the Newton gravitational constant measured in laboratory.
In the absence of direct interaction between DM and DE:

$$T_{\mu\,(DE);\nu}^{\nu} = 0 \;.$$

Possible forms of DE

- Physical DE.
  New non-gravitational field of matter. DE proper place – in the rhs of gravity equations.
- Geometrical DE.
  Modified gravity. DE proper place – in the lhs of gravity equations.
- Λ - intermediate case.

Observations: $T_{\mu\,(DE)}^{\nu}$ is very close to $\Lambda\delta_{\mu}^{\nu}$ for the concrete solution describing our Universe;

$$|< w_{DE} > +1| < 0.1 \ ,$$

where $w_{DE} \equiv p_{DE}/\epsilon_{DE}$.
$w_{DE} > -1$ – normal case,
$w_{DE} < -1$ – phantom case,
$w_{DE} \equiv -1$ – the exact cosmological constant ("vacuum energy").

$f(R)$ gravity – the simplest form of geometrical DE

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R^\mu_\mu \ .$$

The effective energy-momentum tensor of DE in $f(R)$ gravity:

$$8\pi G T^\nu_{\mu\,(DE)} = F'(R)\, R^\nu_\mu - \frac{1}{2} F(R)\delta^\nu_\mu + \left(\nabla_\mu \nabla^\nu - \delta^\nu_\mu \nabla_\gamma \nabla^\gamma\right) F'(R)\,.$$

The most critical test for all $f(R)$ models of present dark energy: anomalous growth of density perturbations in the non-relativistic matter component at recent redshifts $z \sim (1-3)$.

4 fundamental cosmological constants

One-to-one relation to the 4 cosmological stages.
A fundamental theory beyond each of these constants.

- Characteristic amplitude of primordial scalar (adiabatic) perturbations.

$$\Delta^2_{\mathcal{R}} = 2.4 \times 10^{-9} \ , \quad P_s(k) = \int \frac{\Delta^2_{\mathcal{R}}}{k} dk \ .$$

  Theory of initial conditions – inflation.

- Baryon to photon ratio.

$$\frac{n_b}{n_\gamma} = 6.28 \times 10^{-10} \ \frac{\Omega_b h^2}{0.0023} \ \left(\frac{2.725}{T_\gamma(\mathrm{K})}\right)^3 \ .$$

  Theory of baryogenesis.

- Baryon to total non-relativistic matter density.

$$\frac{\rho_b}{\rho_m} = 0.167 \, \frac{\Omega_b}{0.045} \, \frac{0.27}{\Omega_m} \; .$$

Theory of dark matter.

- Energy density of present dark energy.

$$\rho_{DE} = \frac{\epsilon_{DE}}{c^2} = 6.72 \times 10^{-30} \, \frac{\Omega_{DE}}{0.73} \, \left(\frac{H_0}{70} \right)^2 \; \mathrm{g/cm^3} \; ,$$

$$\frac{G^2 \hbar \epsilon_{DE}}{c^7} = 1.30 \times 10^{-123} \, \frac{\Omega_{DE}}{0.73} \, \left(\frac{H_0}{70} \right)^2 \; .$$

Theory of present dark energy (of a cosmological constant).

The minimal present standard cosmological model

ΛCDM $+$ $(\mathcal{K} = 0)$+(scale-invariant adiabatic perturbations) contains two more parameters:

- H_0 – not a constant, but a present value of $H(t)$;
- τ – optical width after recombination – a constant, but not fundamental.

4 fundamental cosmological constants $\Longrightarrow$ no more than 4 cosmological "coincidences", all other "coincidences" exist already at the level of usual laboratory physics.

New fundamental constants from cosmology soon?

In case of primordial dark energy (inflation) – yes.

The basic prediction from inflation is that the primordial spectrum of adiabatic scalar perturbations is generically only approximately flat (i.e. Harrison-Zeldovich like), namely, $n_s - 1$ is a small and slowly varying function of scale, apart from one exceptional and very specific form of an inflaton potential: $V(\phi) \propto \phi^{-2}$ in the slow-roll approximation, see Starobinsky (2005) for the exact solution for $V(\phi)$ beyond slow-roll.

$$n_s(k) - 1 = \frac{d \ln P_s(k)}{d \ln k}$$

WMAP has got some first evidence for a non-zero value of this function: $< n_s > -1 \approx -0.04$ at 3σ level, but only under the additional assumption that the tensor-to-scalar ratio $r = 0$. Thus, no definite statement about the value of $< n_s > -1$ may be made at present. But this quantity will be certainly measured with 10% accuracy by Planck in no more than 2-3 years. So, at least one new fundamental constant is guaranteed to be be measured soon. More optimistically, the constant r and primordial gravitational waves may be discovered, too.

Keeping the same number of fundamental constants

Observations tend to increase the number of fundamental constants, but theory can counteract it by "unification", by expressing these new constants through already existing ones. Inflationary theory can keep the number of fundamental cosmological constants equal to 4, since there exists viable one-parameter inflationary models. Incidentally, they are also the pioneering ones.

- Geometrical primordial dark energy – the $f(R) = R + R^2/(6M^2)$ inflationary model (Starobinsky, 1980).
  $M = 3.0 \times 10^{-6}\ (50/N)\ M_{Pl}$ – from $\Delta_{\mathcal{R}}$. Then
  $n_s - 1 = -2/N = -0.04\ (50/N)$,
  $r = 12/N^2 = 0.005\ (50/N)^2$,
  r is the tensor-to-scalar ratio for primordial perturbations.

- Scalar field inflation with a phase transition – the "new" inflationary model (Linde, 1982; Albrecht and Steinhardt, 1982) with $V(\phi) = V_0 - \lambda\phi^4/4$.
  $\lambda = 2.8 \times 10^{-13}\ (50/N)^{3/2}$ – from $\Delta_{\mathcal{R}}$. Then
  $n_s - 1 = -3/N = -0.06\ (50/N)\ , \quad r \ll 1\,.$
- Scalar field inflation without a phase transition – the simplest chaotic inflationary model with $V(\phi) = m^2\phi^2/2$ (Linde, 1983, though its mathematical solution was known since Starobinsky, 1978, where this model was used for a bouncing universe scenario).
  $m = 1.4 \times 10^{-6}\ (50/N)\ M_{Pl}$ – from $\Delta_{\mathcal{R}}$. Then
  $n_s - 1 = -2/N = -0.04\ (50/N)\ ,$
  $r = 8/N = 0.16\ (50/N)\ .$
  The present 95% CL upper bound: $r < 0.24$.
  No more than one of these models may remain after the Planck data.

Homage to Oskar Klein

The derivation of the primordial perturbation spectra is directly based on two pioneer contributions by Oscar Klein:

I. The Klein-Gordon equation (or, more correct historically, the Fock-Klein-Gordon equation).
Equations for scalar and tensor perturbations on a Friedmann-Robertson-Walker background can be reduced to this equation with some effective mass (zero mass in the case of tensor perturbations - gravitational waves).

II. The Klein paradox, generalized to the boson case and interpreted as particle creation by external fields (gravitational in this case). In turn, it takes the form of generation of metric perturbations in the cosmological situation (in the super-Hubble regime $\lambda \equiv a(t)/k \gg H^{-1}(t)$).

Remarks.

1. The use of the effect of gravitational particle creation to produce initial metric perturbations is not restricted to the inflationary scenario and, in fact, is used in almost all its existing alternatives, too.

2. At this place, a large and completely independent area of research is hidden: quantum-to-classical transition for these metric perturbations. Shortly, the perturbations remains quantum even now but they are practically indistinguishable from classical (c-number) stochastic quantities with an almost Gaussian statistics.

Conclusions

- At present, cosmology requires the introduction of 4 fundamental constants to describe observational data, additional to those known from laboratory physics.
- At least one new fundamental constant will be discovered in the next 2-3 years, but the theory is ready to express it through the existing constants.
- Regarding the present dark energy:
  a) still no statistically significant deviation from an exact cosmological constant;
  b) one constant is sufficient to describe its properties;
  c) no more than one new "coincidence problem".
- Regarding the primordial dark energy driving inflation in the early Universe:
  a number of inflationary models having only one free parameter can explain all existing observational data.