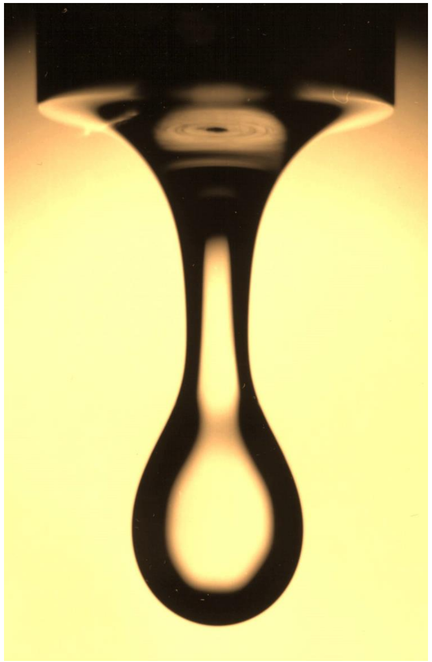
Topological transitions and singularities in fluids: Life of a Drop

Manne Siegbahn Memorial Lecture, Oct. 18, 2007



During its brief existence, drop goes through many stages.

Shape changes often accompanied by dynamic singularities*.

*Dynamic singularities: infinitesimally small, very short time

Test-bed for understanding broad class of phenomena.

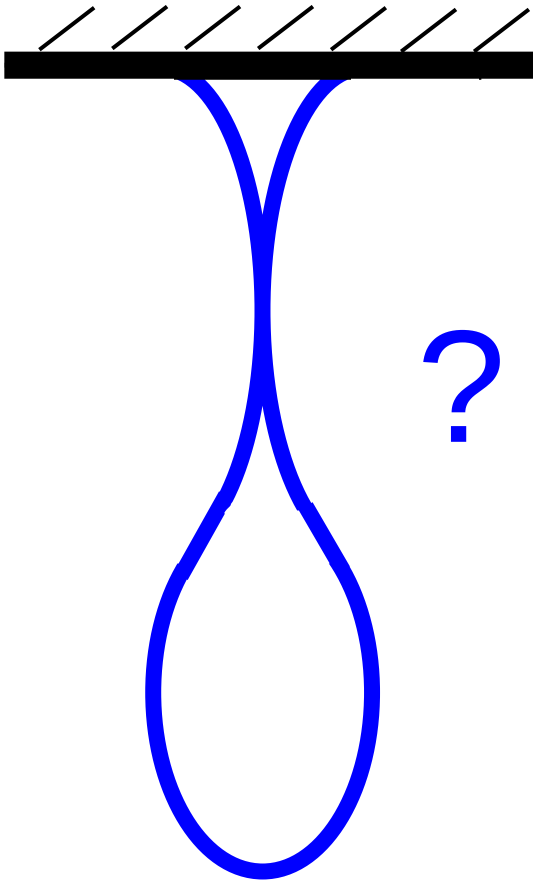
How do drops fall, break apart?

Topology changes - real transition

Neck radius $\rightarrow$ 0 (curvature $\rightarrow \infty$)

Pressure $\rightarrow \infty$

Cannot do simulation to get to other side of snapoff



Is there understanding for these transitions like that for thermodynamic phase transitions?

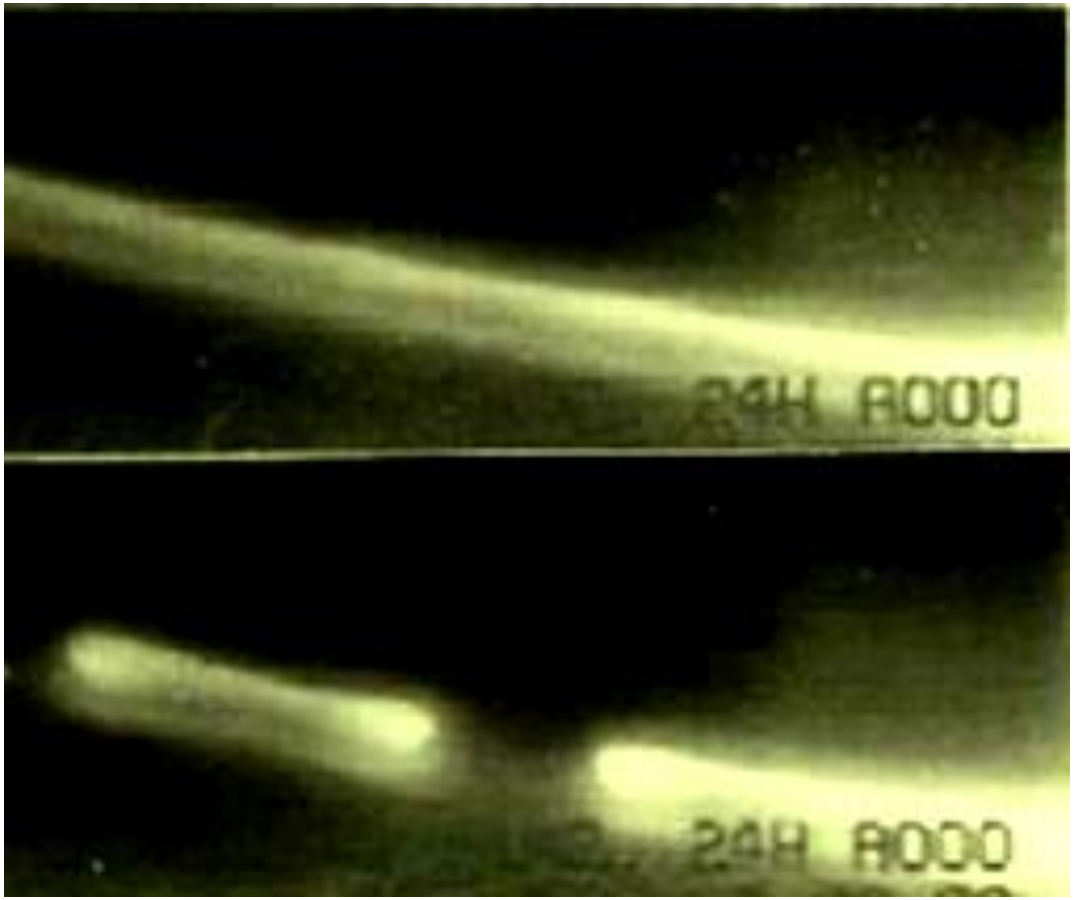
Similar behavior: Star formation

PILLARS OF CREATION
IN STAR-FORMING REGION

Gas Pillars in M16 - Eagle Nebula

Hubble Space Telescope 4/1/95

Similar behavior: Breakup of bacteria colonies



Elena Budrene - Harvard

Dynamic singularities appear everywhere in physics -

celestial → microscopic → nuclear fission…

Who Did It

Experiment

Itai Cohen
Nathan Keim
Xiangdong Shi
Lei Xu

Theory/Simulation

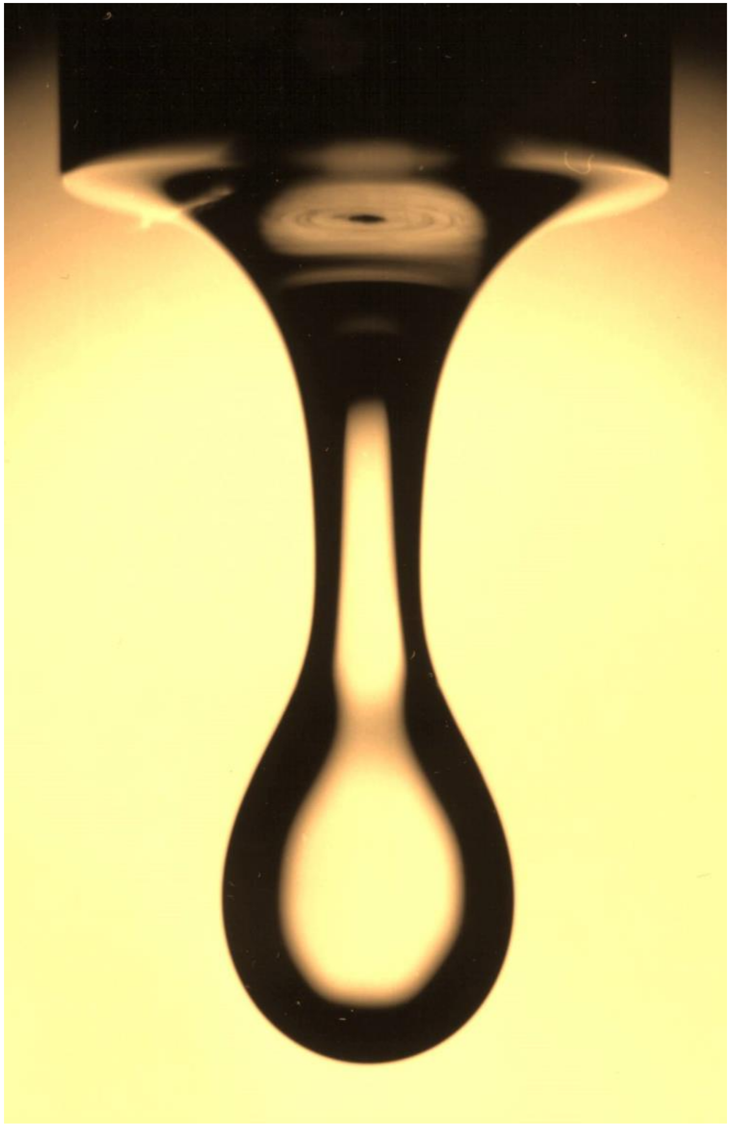
Osman Basaran
Michael Brenner
Pankaj Doshi
Jens Eggers
Laura Schmidt
Wendy Zhang

Funding: NSF

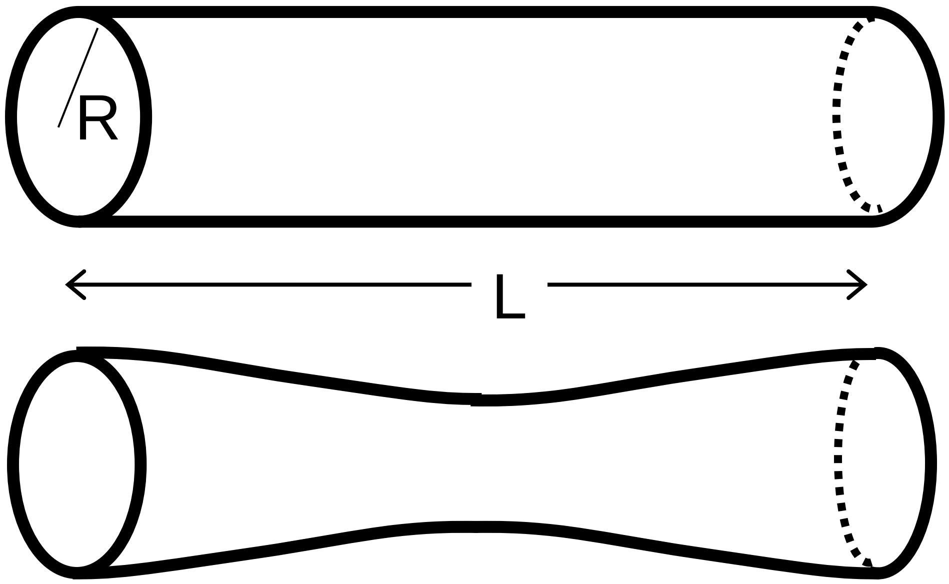
Birth and Childhood

A happy childhood!

Surface tension + gravity



Rayleigh-Plateau Instability



Surface area *decreases* if $L > 2\pi R$.
Unstable to perturbations.

Pressure greatest at minimum thickness
$\Rightarrow$ liquid squeezed out.

Bolas Spider

Midlife Crisis

Midlife Crisis

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are needed to see this picture.

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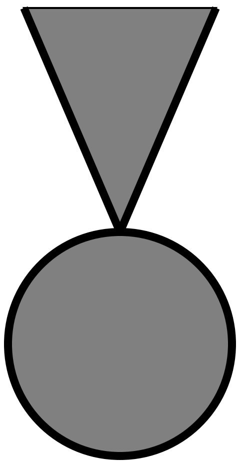
Water into Air



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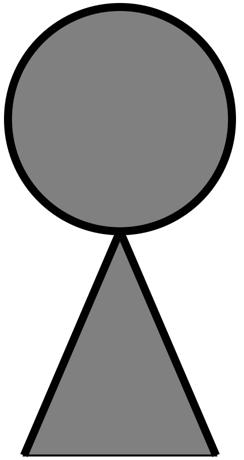

Singularity is same even though gravity points in opposite direction

Xiangdong Shi and Michael Brenner

Water Drops

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3ivx D4 4.5.1 decompressor
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How to Think About Shapes: *Scale invariance*

(borrowed from statistical mechanics)

Breakup $\Longrightarrow$ radius smaller than any other length.

Dynamics insensitive to *all* other lengths.

Flow depends *only* on shrinking radius.

How to Think About Shapes: *Scale invariance*

(borrowed from statistical mechanics)

Breakup $\Rightarrow$ radius smaller than any other length.

Dynamics insensitive to *all* other lengths.

Flow depends ***only*** on shrinking radius.

But: Radius depends on flow
(which depends on radius
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. . .

Self-similar structure:

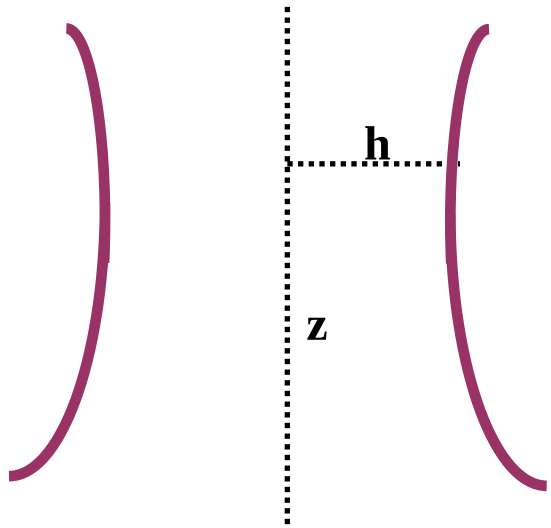
Blow up any part $\Rightarrow$ regain original.

Universal shapes

Similarity Solution

$$h(z,t) = f(t)\, H[(z-z_o)/f(t)^{\beta}]$$

Similarity solutions same AT DIFFERENT TIMES with different magnifications along h and z.



Scaling determined by force balance at singularity.

PDE $\Rightarrow$ ODE

Keller and Miksis (83); Eggers, RMP (97)

Unhappy drops are all unhappy in their own way

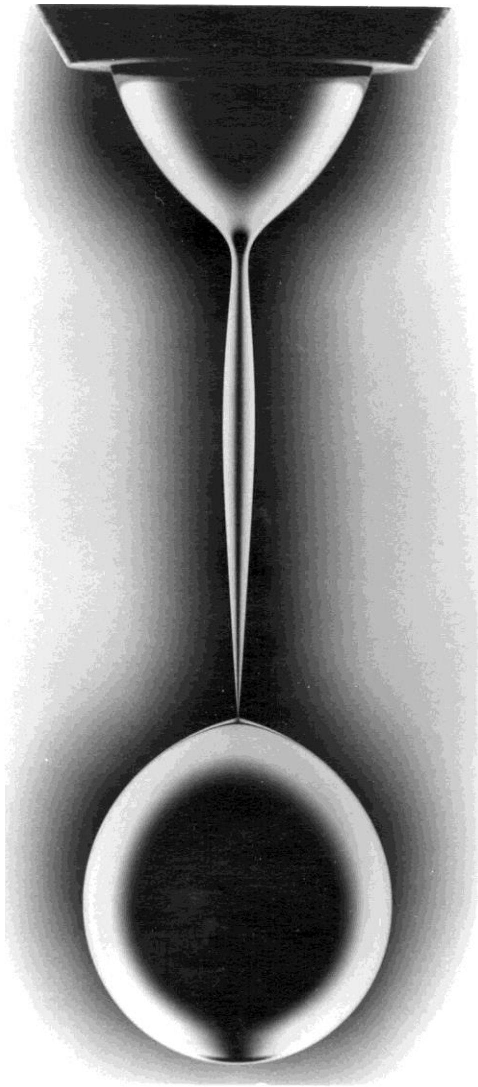
Explore different asymptotic regimes by tuning parameters.

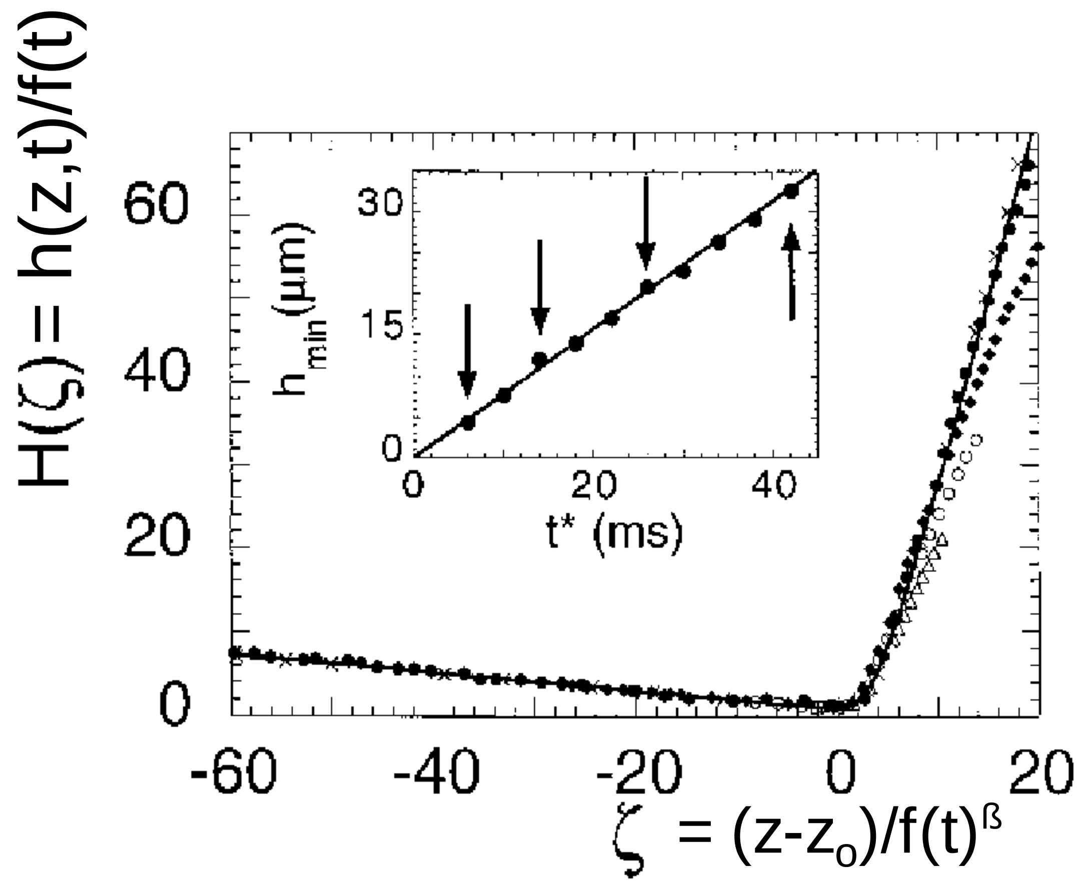
Depends on:

- viscosity of inner fluid, μ
- viscosity ratio of fluids, λ (air is a fluid)
- density of inner fluid, ρ
- density difference, $\Delta\rho$
- surface tension, γ
- nozzle diameter, D

Scaling Profiles - Glycerol into Oil

Stretching axes by different amounts at different times produces master curve





Itai Cohen, Michael Brenner
Jens Eggers, Wendy Zhang

Singularities tame non-linearity of Navier-Stokes Eqs.

Role of *scale invariance -*

borrowed from critical phenomena

Near singularity, dynamics insensitive to all other lengths

Emphasize what is *Universal*

But . . .

Water into Oil

Not so simple

Persistence of memory.

No similarity solution.

No universality!

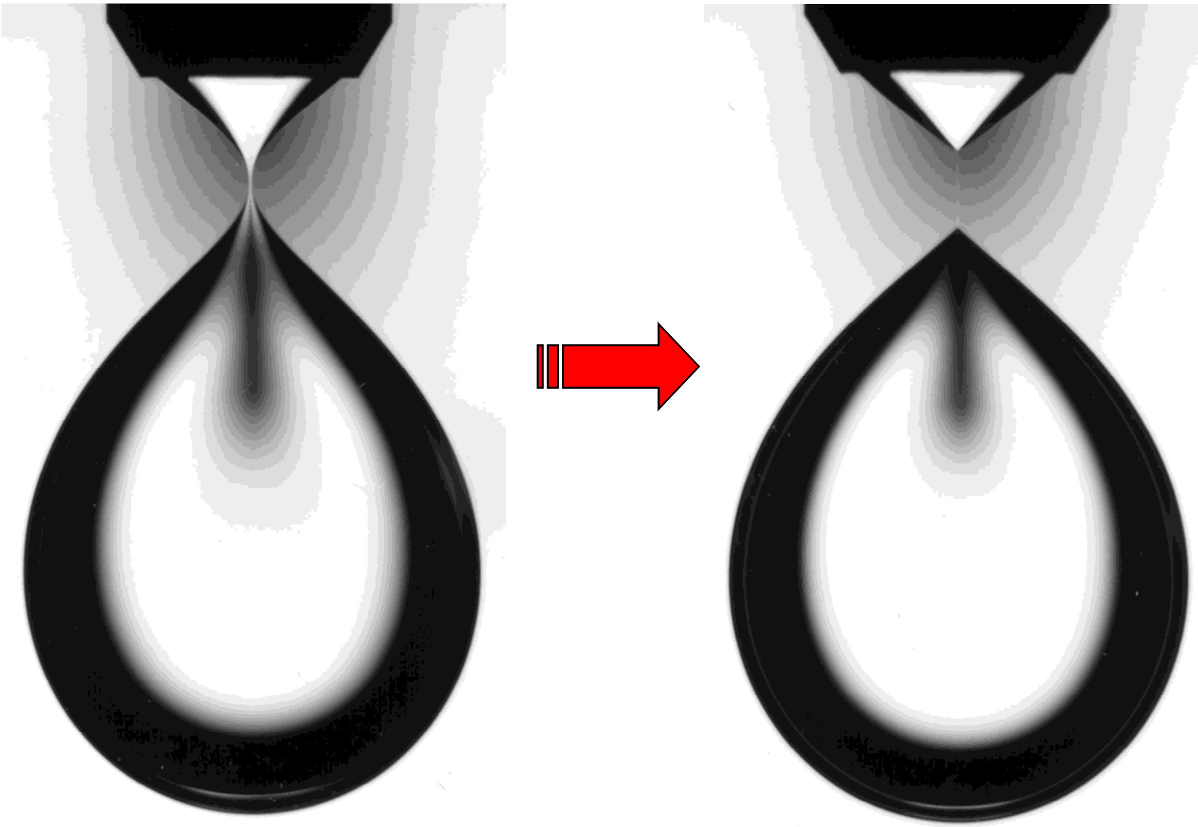
Separation of scales but also of axial and radial

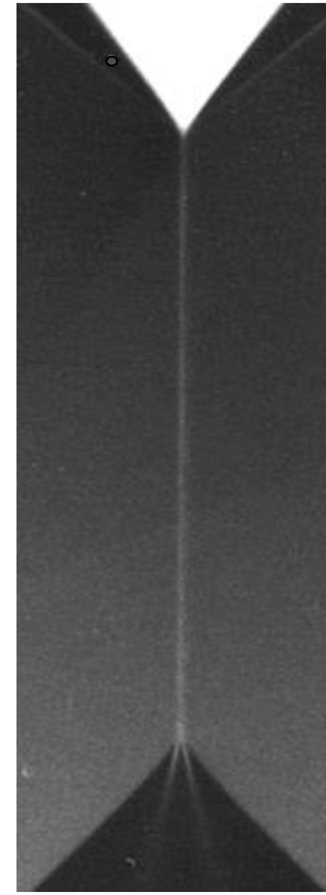
length scales

I. Cohen, W. Zhang
P. Doshi, O. Basaran
P. Howell, M. Siegel

Water into Oil

Continued…





Viscosity of water begins to matter

⇒ creates very fine thread.

Remember water drop in air?

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are needed to see this picture.

Remember water drop in air?

What about air drop in water (*i.e., a* bubble)?

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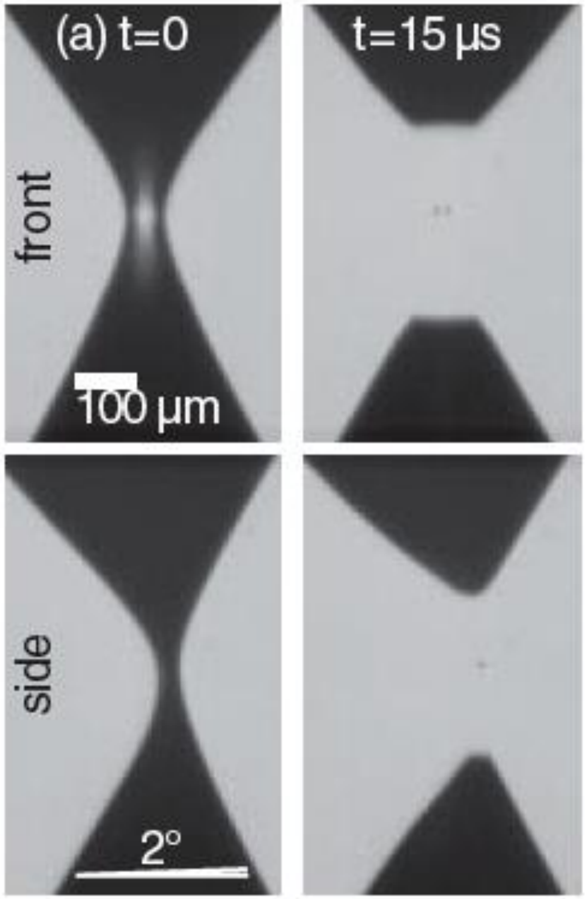
QuickTime™ and a
Cinepak decompressor
are needed to see this picture.

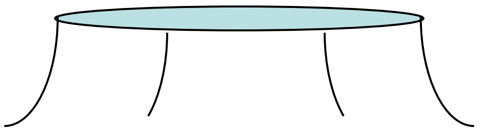
N. Keim, W. Zhang

Perturbations

slight nozzle tilt



oblong nozzle



QuickTime™ and a
Cinepak decompressor
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N. Keim, W. Zhang

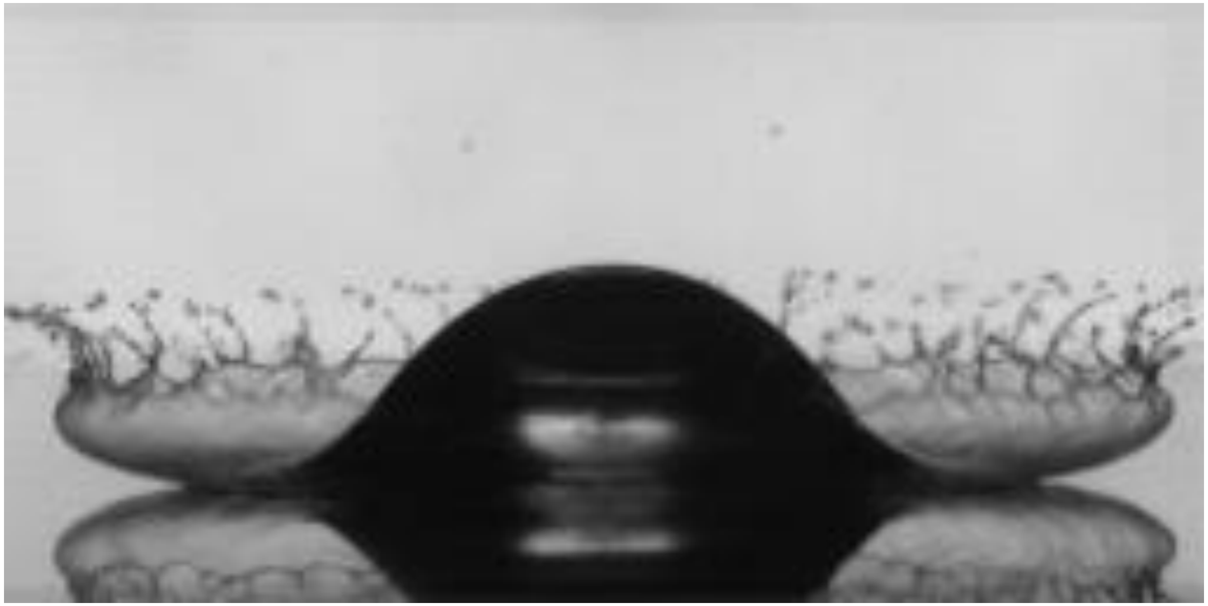
The drop falls $\Rightarrow$ splashes

Is splash interesting?

Break-up localizes energy from the kinetic energy into singular points as surface ruptures.

How?

Coronal splash



Lei Xu, Wendy Zhang

Drop splashes

QuickTime™ and a
decompressor
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Drop of alcohol hitting smooth, dry slide

Lei Xu, Wendy Zhang

Drop splashes

QuickTime™ and a
decompressor
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atmospheric pressure

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decompressor
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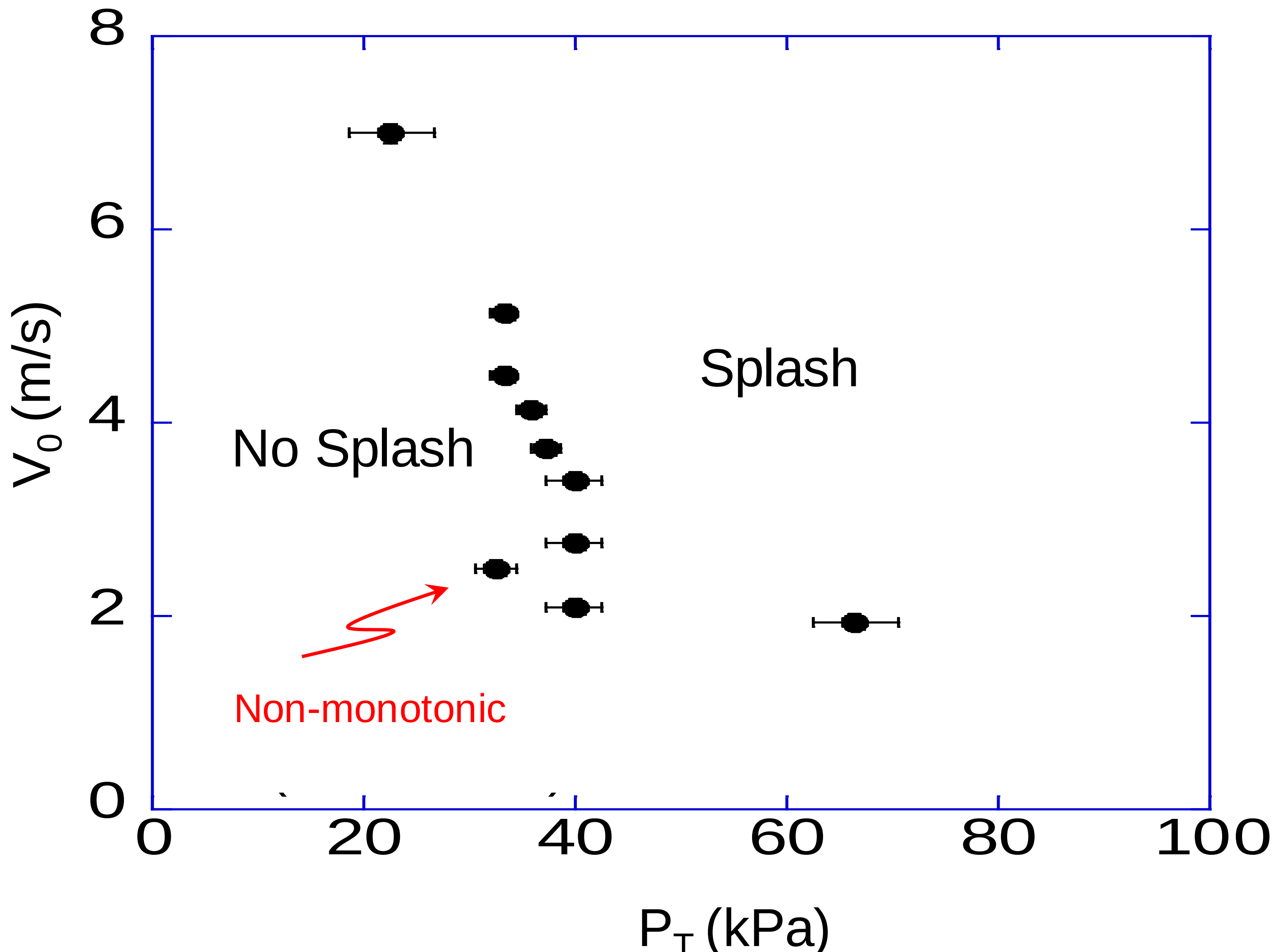
1/3 atmospheric pressure

(Mt. Everest)

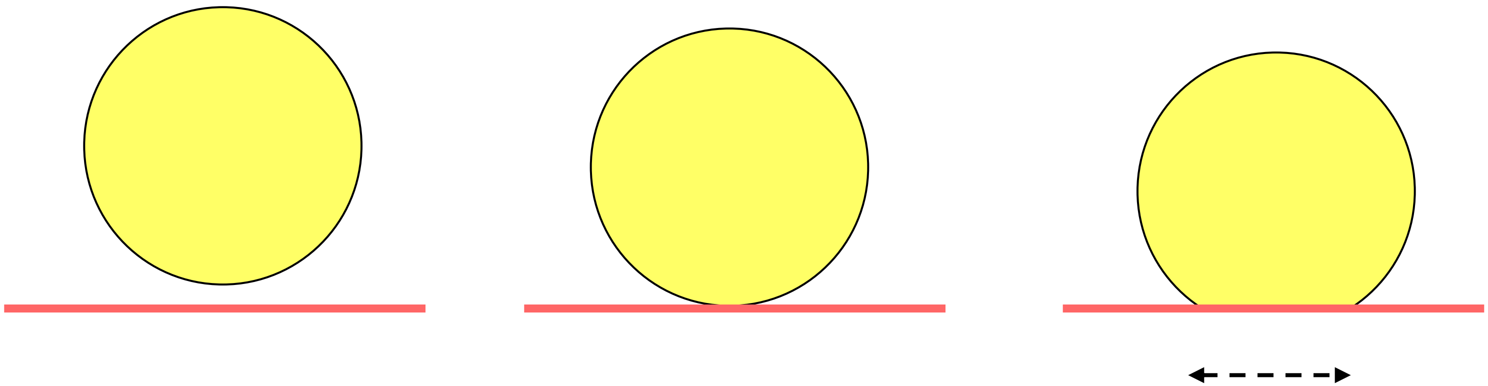
Lei Xu, Wendy Zhang

Impact Velocity *vs.* Threshold Pressure



Lei Xu, Wendy Zhang

Singularity during splash



t_0 $V_{exp} \propto (t - t_0)^{-0.5}$

Drop rim expands infinitely rapidly at moment of impact.

At high viscosity, does air matter?

5 cSt

QuickTime™ and a
decompressor
are needed to see this picture.

High pressure

Low pressure

QuickTime™ and a
decompressor
are needed to see this picture.

Lei Xu, Casey Stevens,
Nathan Keim

At high viscosity, does air matter?

10 cSt

QuickTime™ and a
Cinepak decompressor
are needed to see this picture.

100 kPa

43 kPa

QuickTime™ and a
Cinepak decompressor
are needed to see this picture.

Air still matters.

Does compressibility?

1000 cSt

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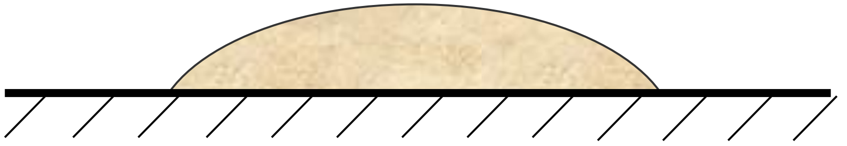
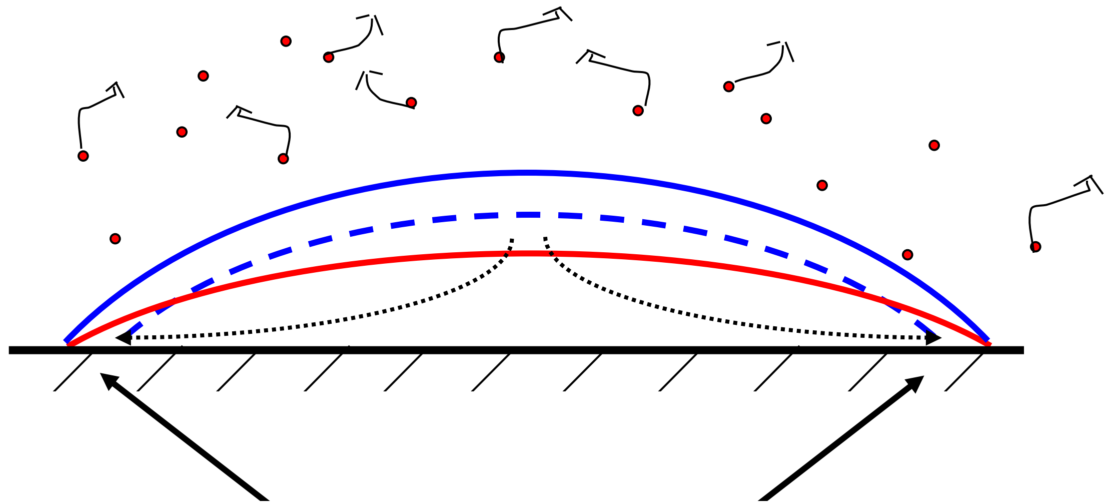
Last stage of the drop: What remains?



Why are drops always ring-shaped?

How does evaporation bring everything to edge?



Drop pinned at contact line

Analogy with Electrostatics

$$\nabla^2 \phi = 0$$ (steady state diffusion)

$$\phi = \phi_o$$ vapor saturated at surface

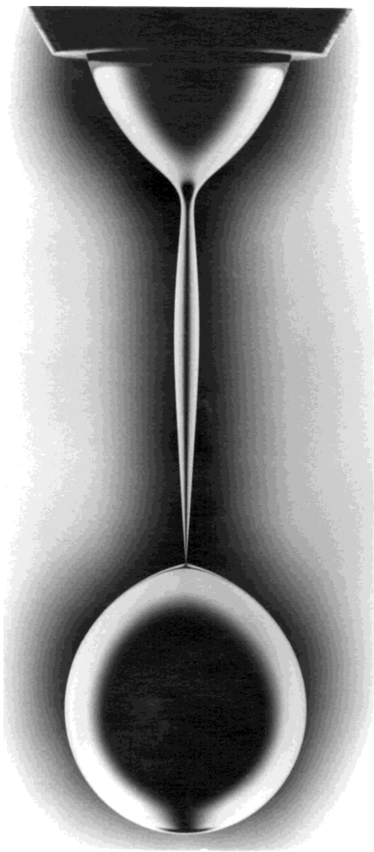
Equations for potential of a charged conductor:
at points, electric field (evaporation rate) diverges.

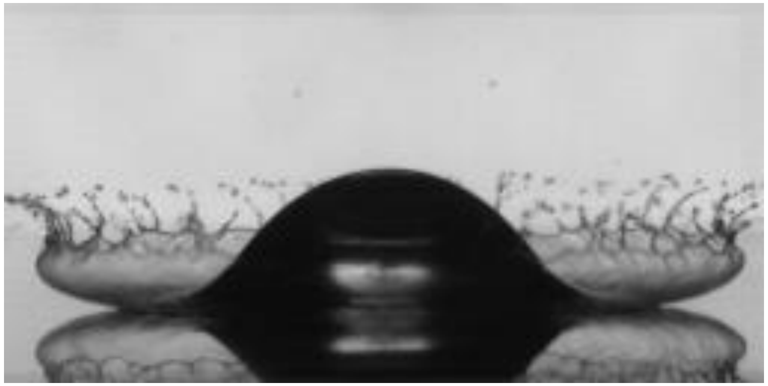
Every stage of drop's life arouses astonishment.

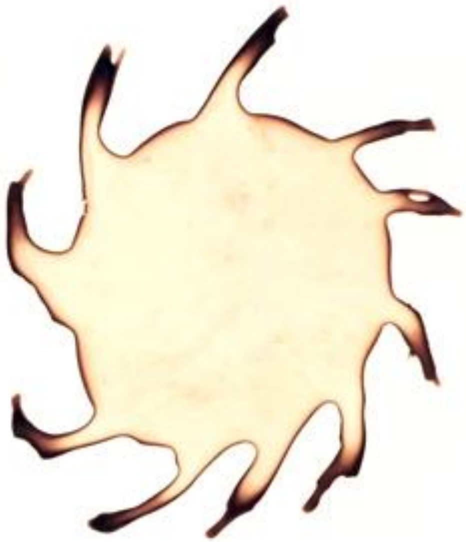
Ideas used to treat singularities appear in different variations - from thermodynamic to topological transitions

A great idea "is like a phantom ocean beating upon the shores of human life in successive waves of specialization."

A. N. Whitehead







Surprises and beauty await us…

even in the most familiar phenomena!

Navier-Stokes Equation

$$\rho[\partial \mathbf{v}/\partial t + (\mathbf{v} \cdot \boldsymbol{\nabla})]\mathbf{v} = -\boldsymbol{\nabla} P + \eta \nabla^2 \mathbf{v} + \mathbf{F}$$

inertial terms = internal pressure + viscous force + body force

+ Incompressibility Equation

+ Laplace pressure equation for surface

Remember water drop in air?

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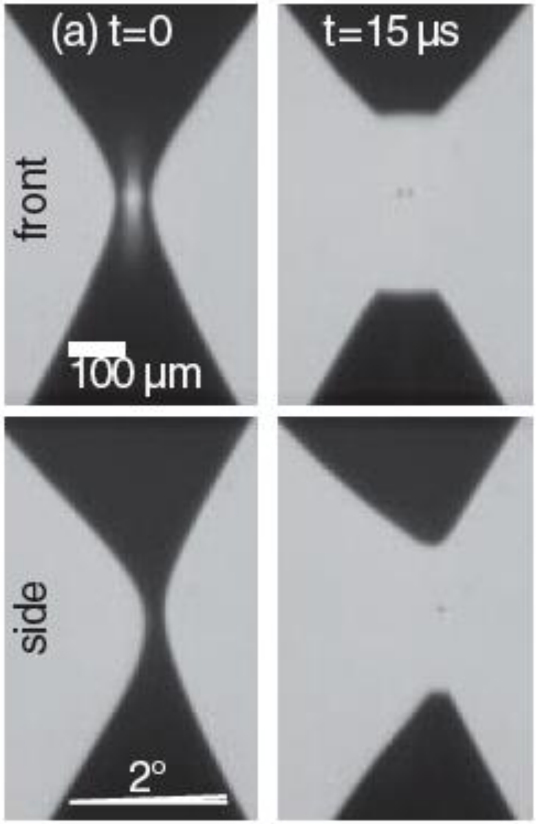
What about air drop in water (*i.e., a* bubble)?



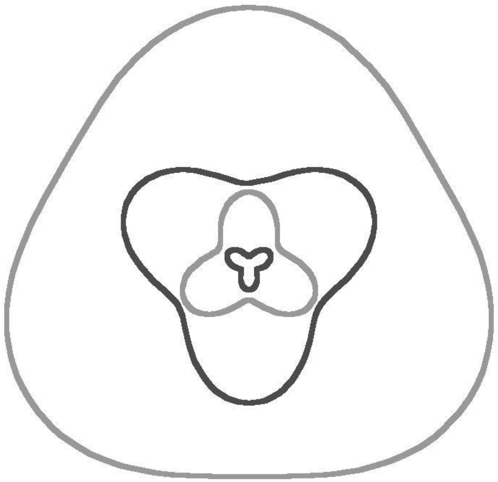
slight nozzle tilt

Singularity sensitive to small perturbations - remembers axial asymmetry

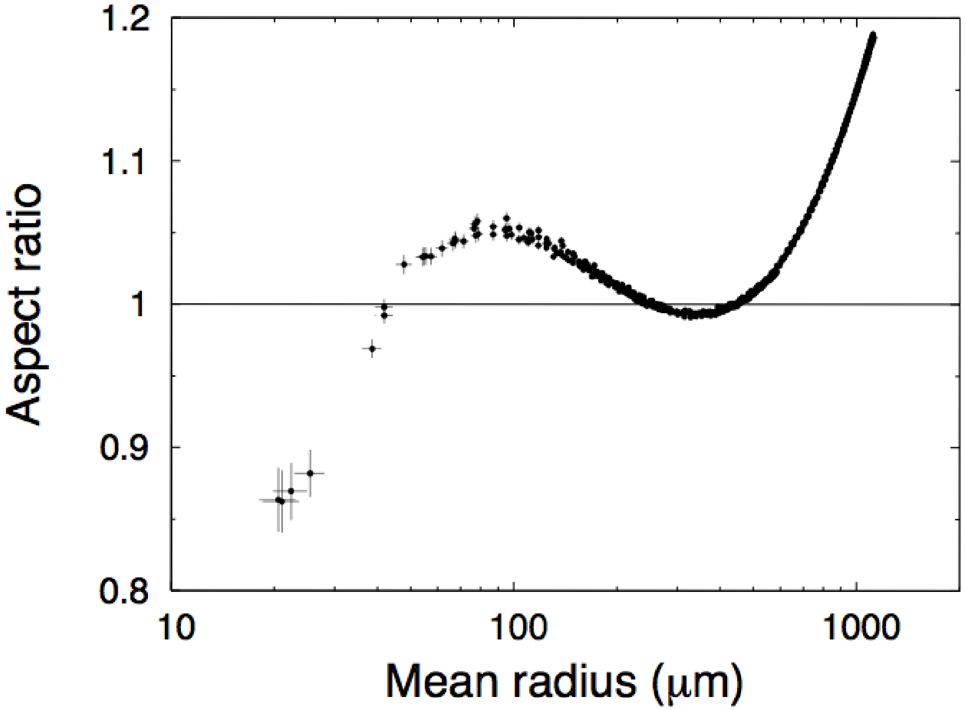
N. Keim, W. Zhang

Memory of all initial amplitudes

Modes oscillate: $a_n(\tau) \propto e^{\pm i c_n \ln \tau}$

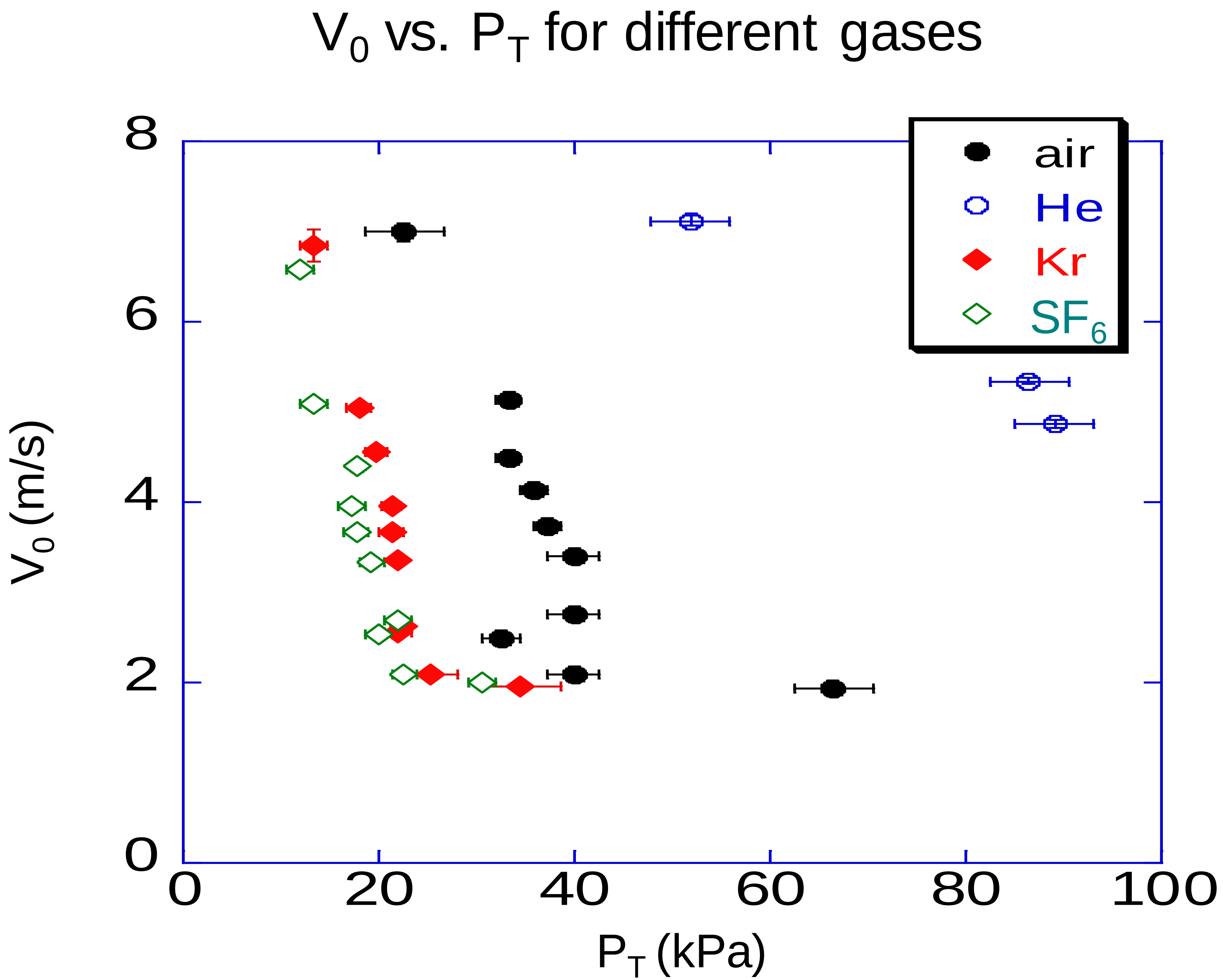


n=3



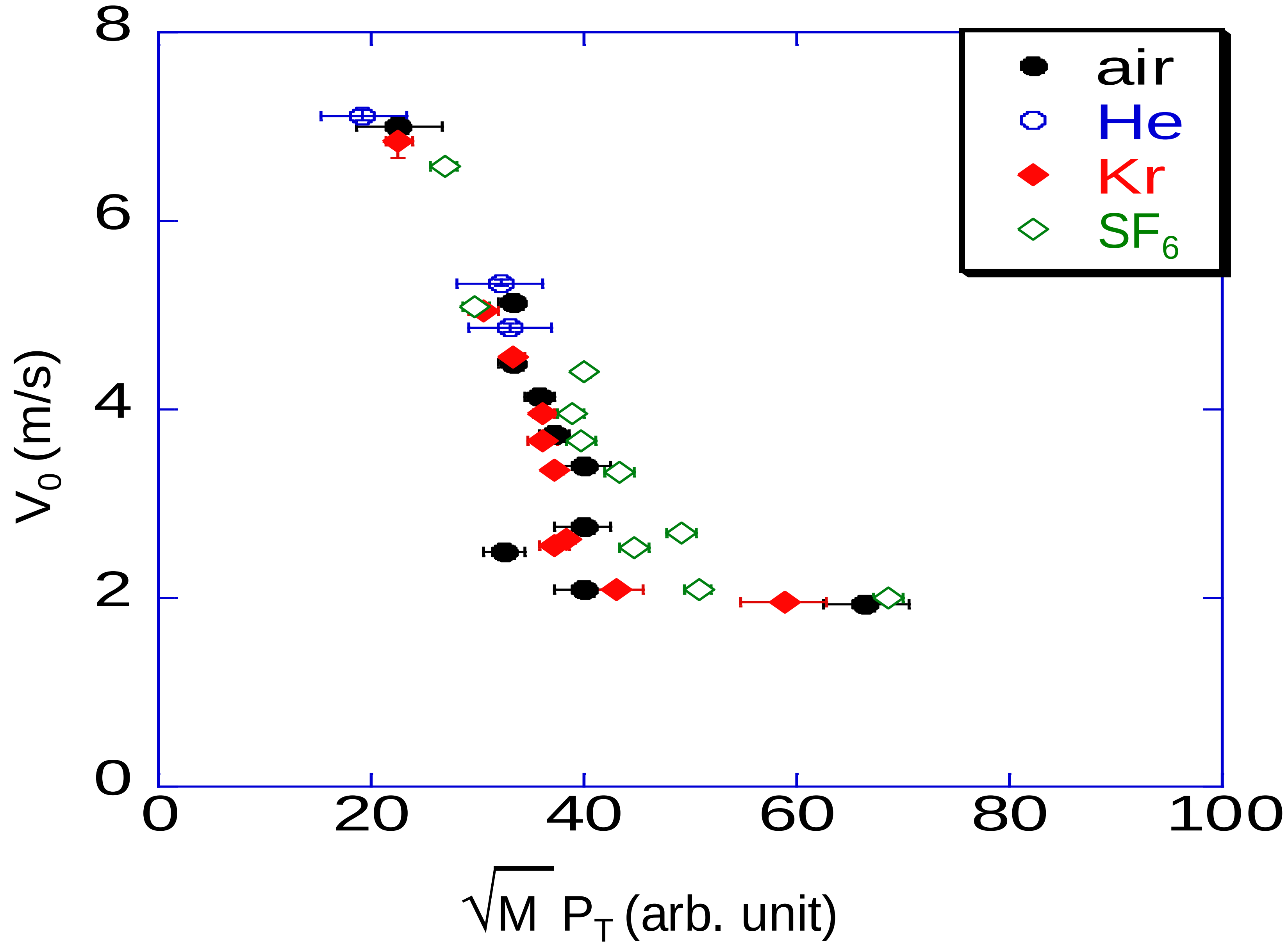
Laura Schmidt & Wendy Zhang

Nathan Keim

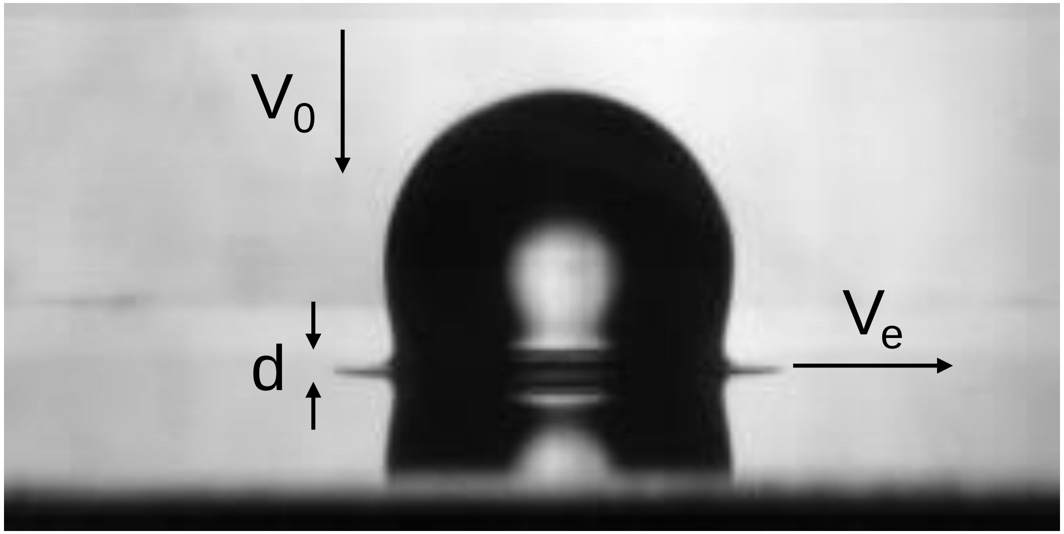


V_0 vs. P_T for different gases
air
He
Kr
SF_6
0
2
4
6
8
0
20
40
60
80
100
V_0 (m/s)
P_T (kPa)

Data Collapse for Different Gases



Model



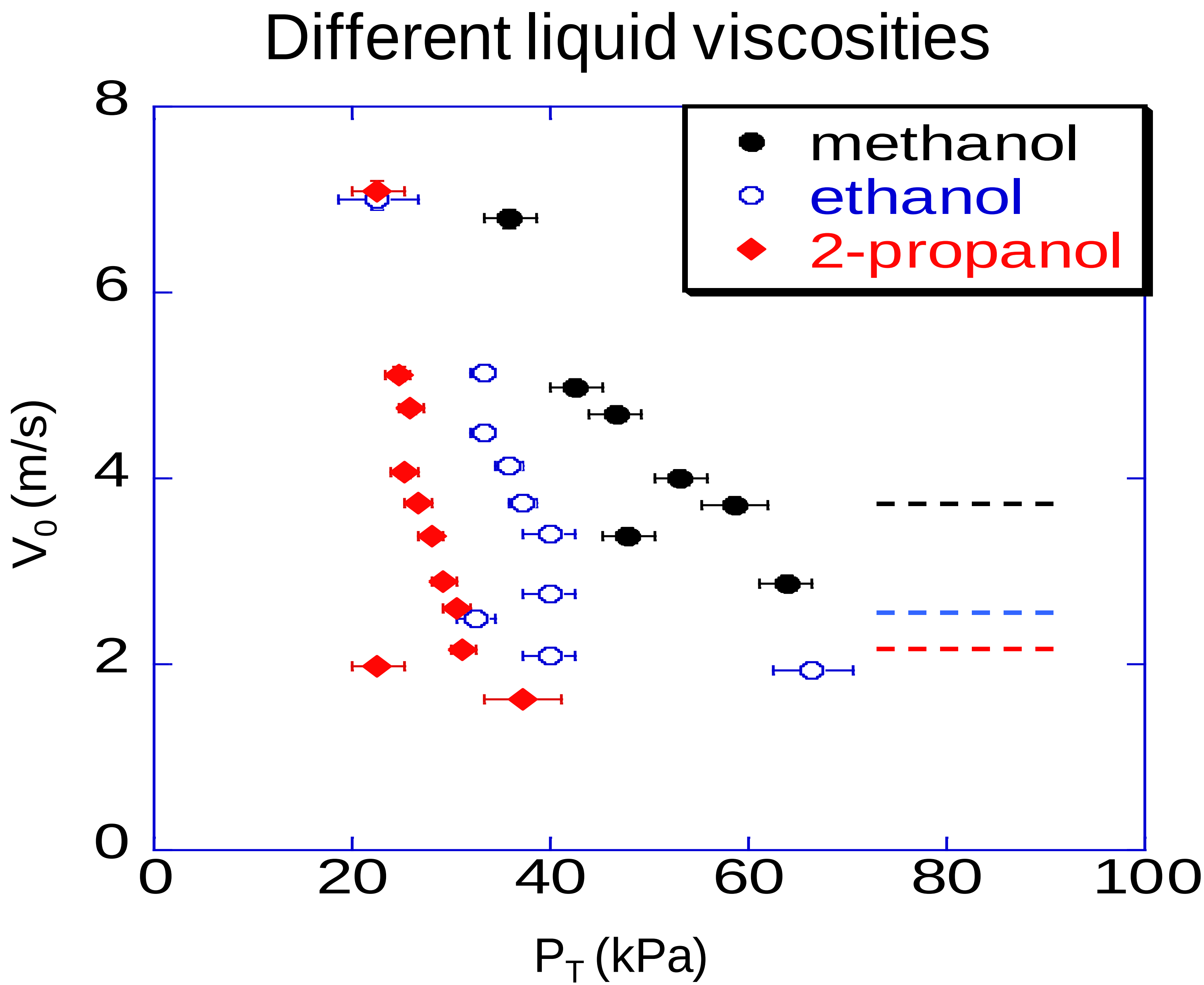
Destabilizing stress: $\Sigma_G \sim \rho_G\, C_G\, V_e$

$$\sim P\, M/kT \quad \sqrt{\gamma kT/M} \quad \sqrt{RV_0/2t}$$

Stabilizing stress: $\Sigma_L = \sigma/d = \sigma/\sqrt{\nu_L t}$

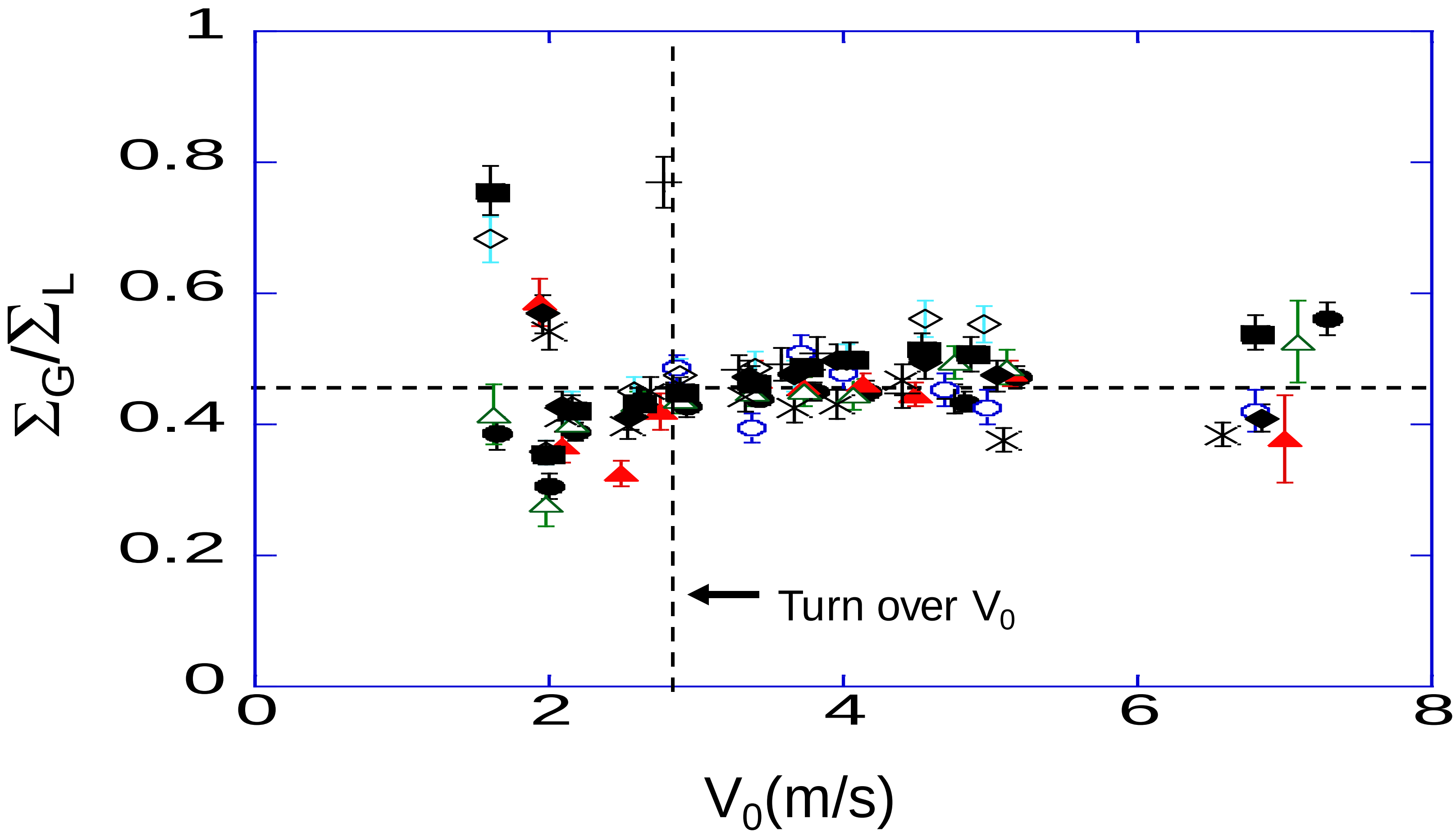
The ratio of Σ_G and Σ_L determines splashing:

$\Sigma_G / \Sigma_L \sim 1$ at threshold



Different liquid viscosities
methanol
ethanol
2-propanol
V0 (m/s)
PT (kPa)
0
2
4
6
8
0
20
40
60
80
100

Threshold values of Σ_G/Σ_L all liquids, gases and velocities

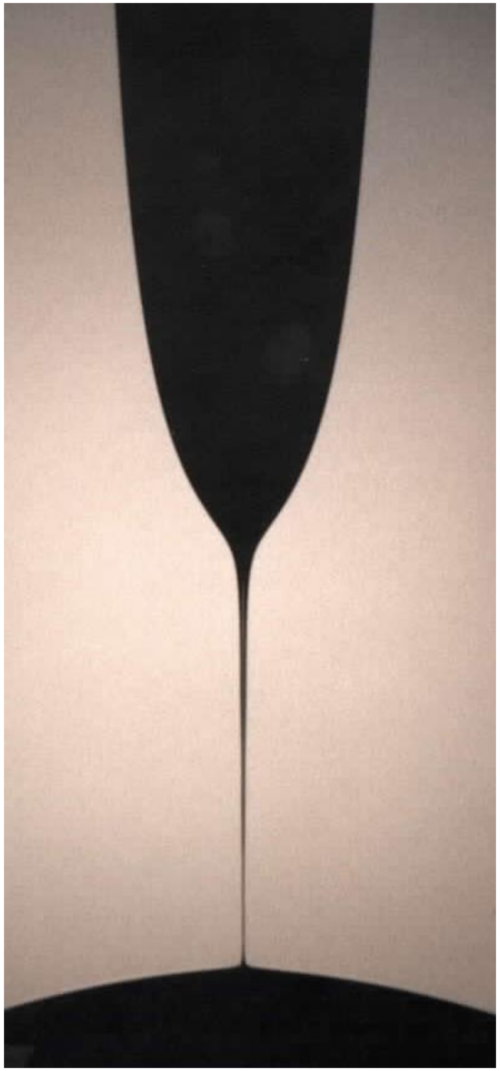


$$\frac{\Sigma_G}{\Sigma_L} = \sqrt{\gamma M P_T} \bullet \sqrt{\frac{R V_0}{2kT}} \bullet \frac{\sqrt{\nu_L}}{\sigma}$$

Glycerol/water

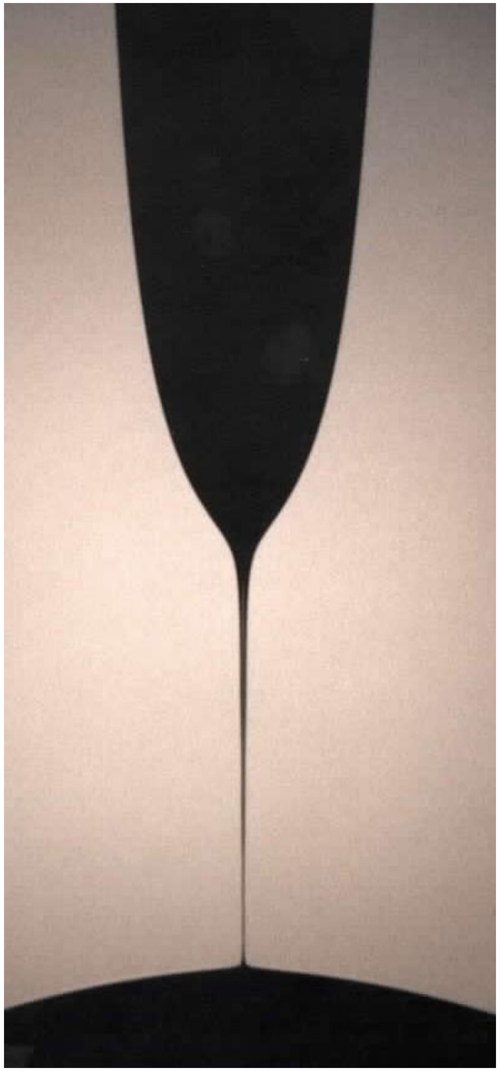
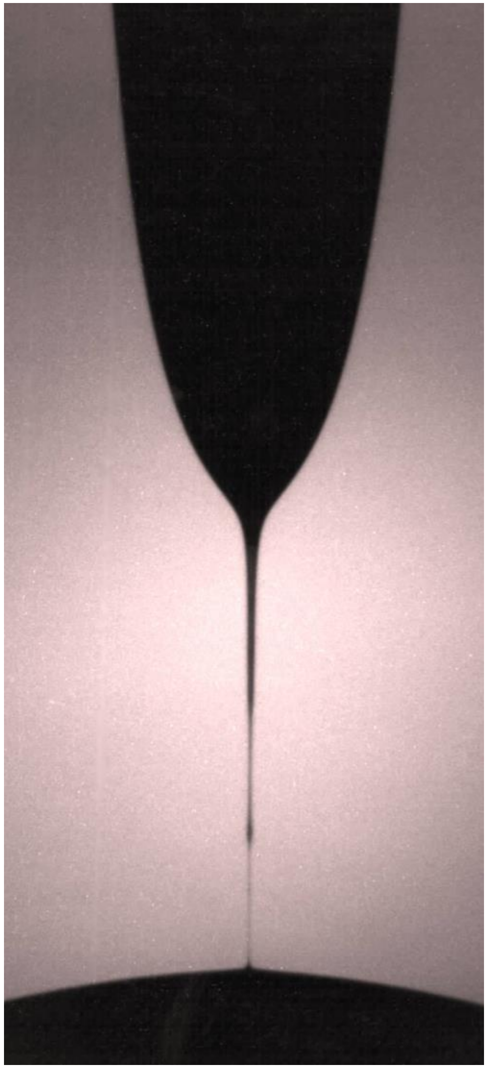
100 x viscosity of water

Glycerol/water



Xiangdong Shi, Michael Brenner

Glycerol/water

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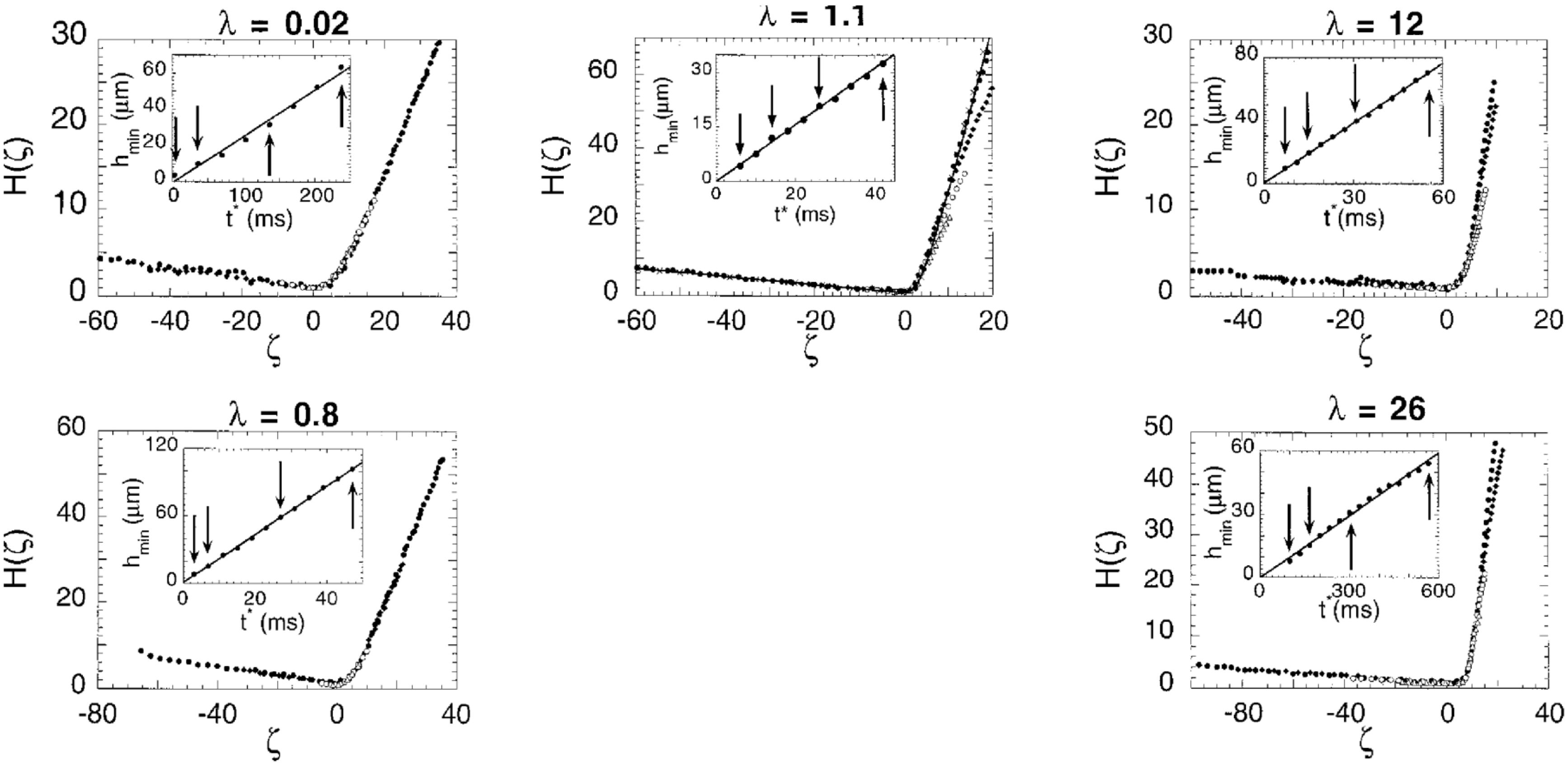
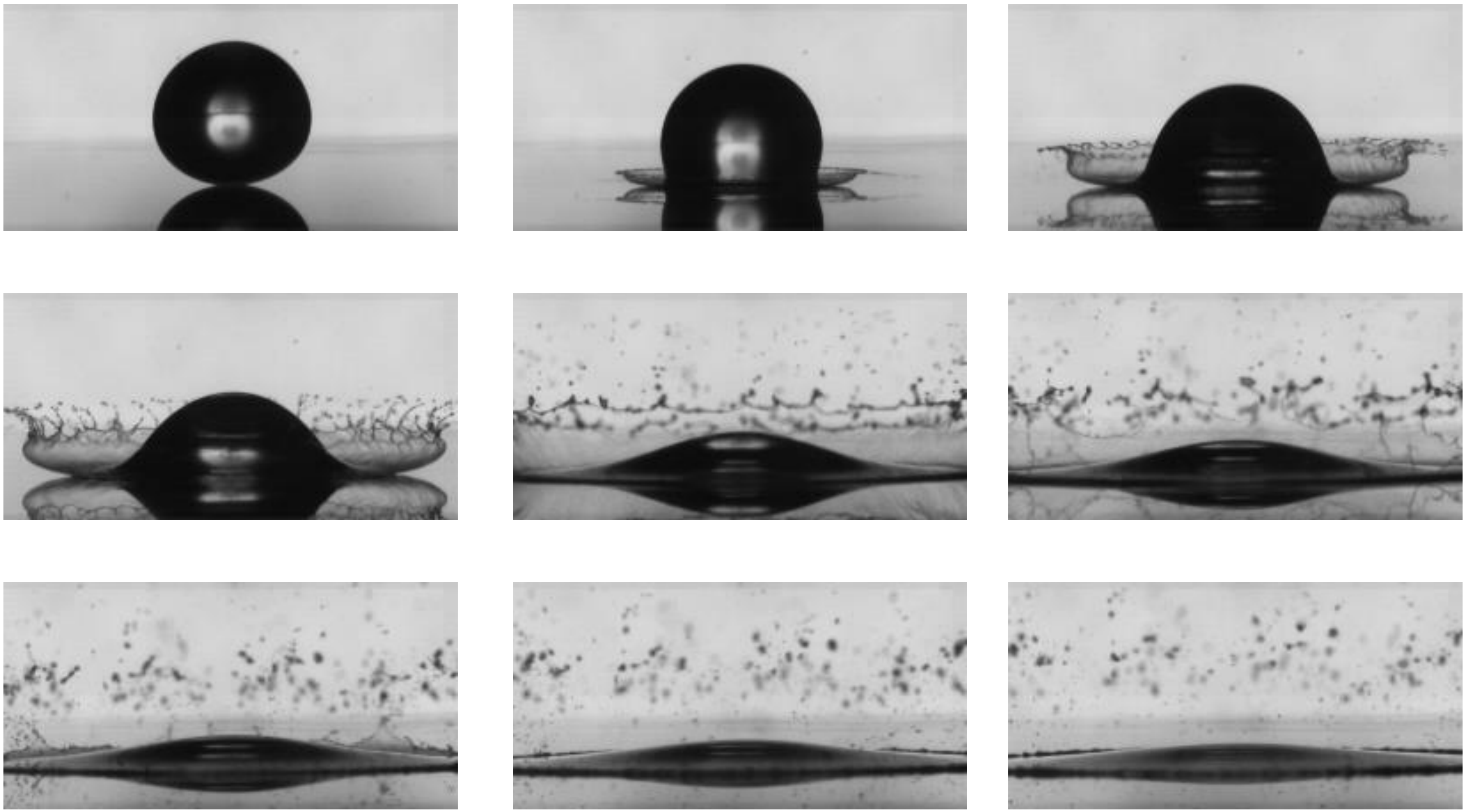
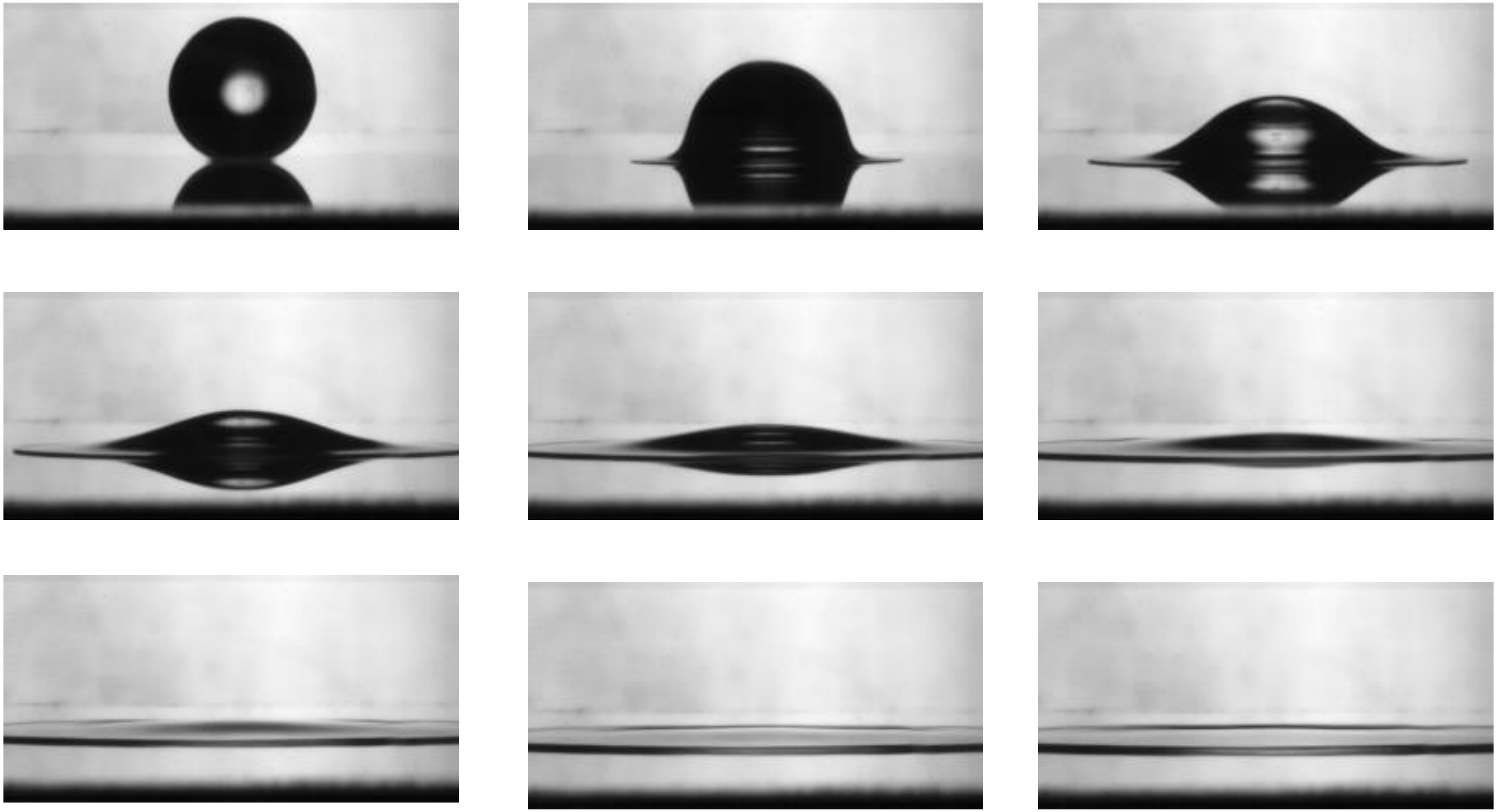
Scaling for different λ



Atmospheric pressure (100kPa)



Reduced pressure (17kPa)

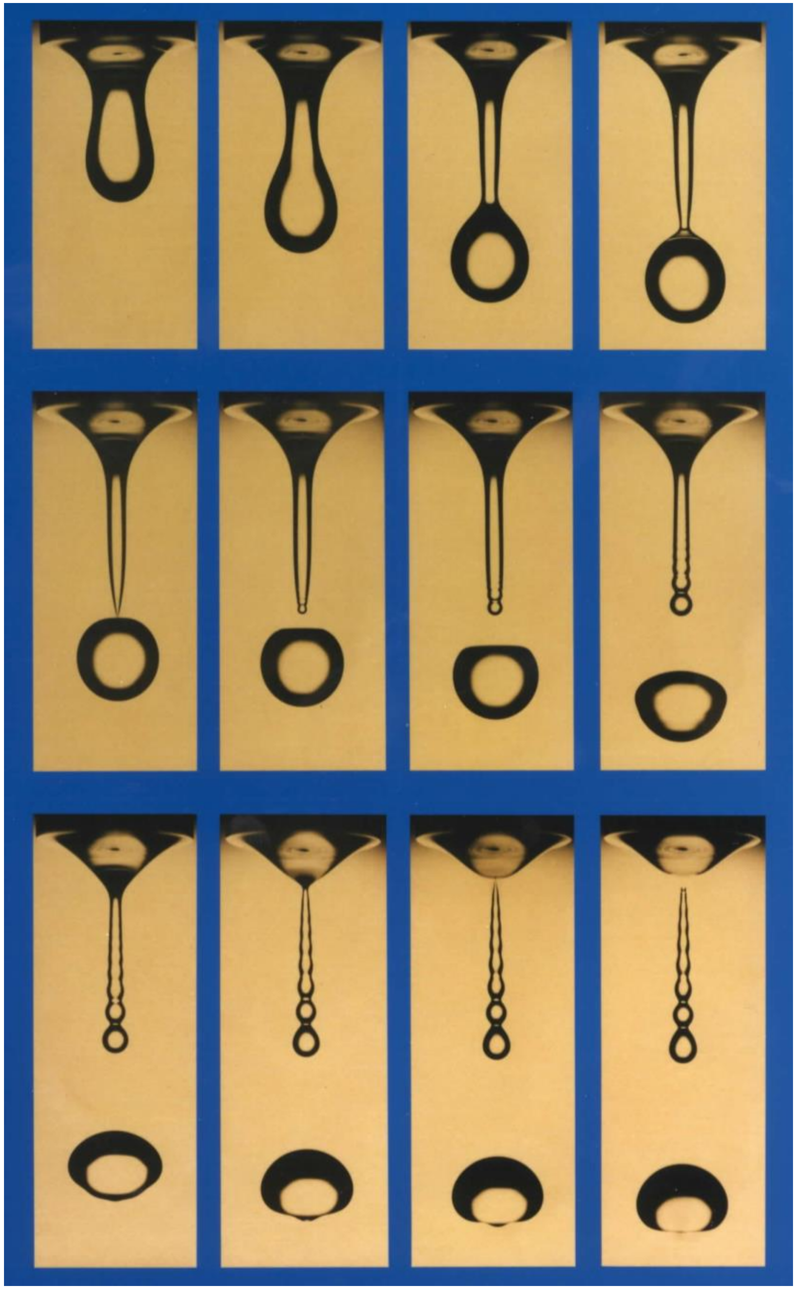


Dynamic singularities appear everywhere in physics - from celestial to microscopic to nuclear fission…

“I am an old man now and when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is turbulent motion of fluids. And about the former I am really rather optimistic.”

Sir Horace Lamb (1932)

Water falling in air



NOT like cartoon!

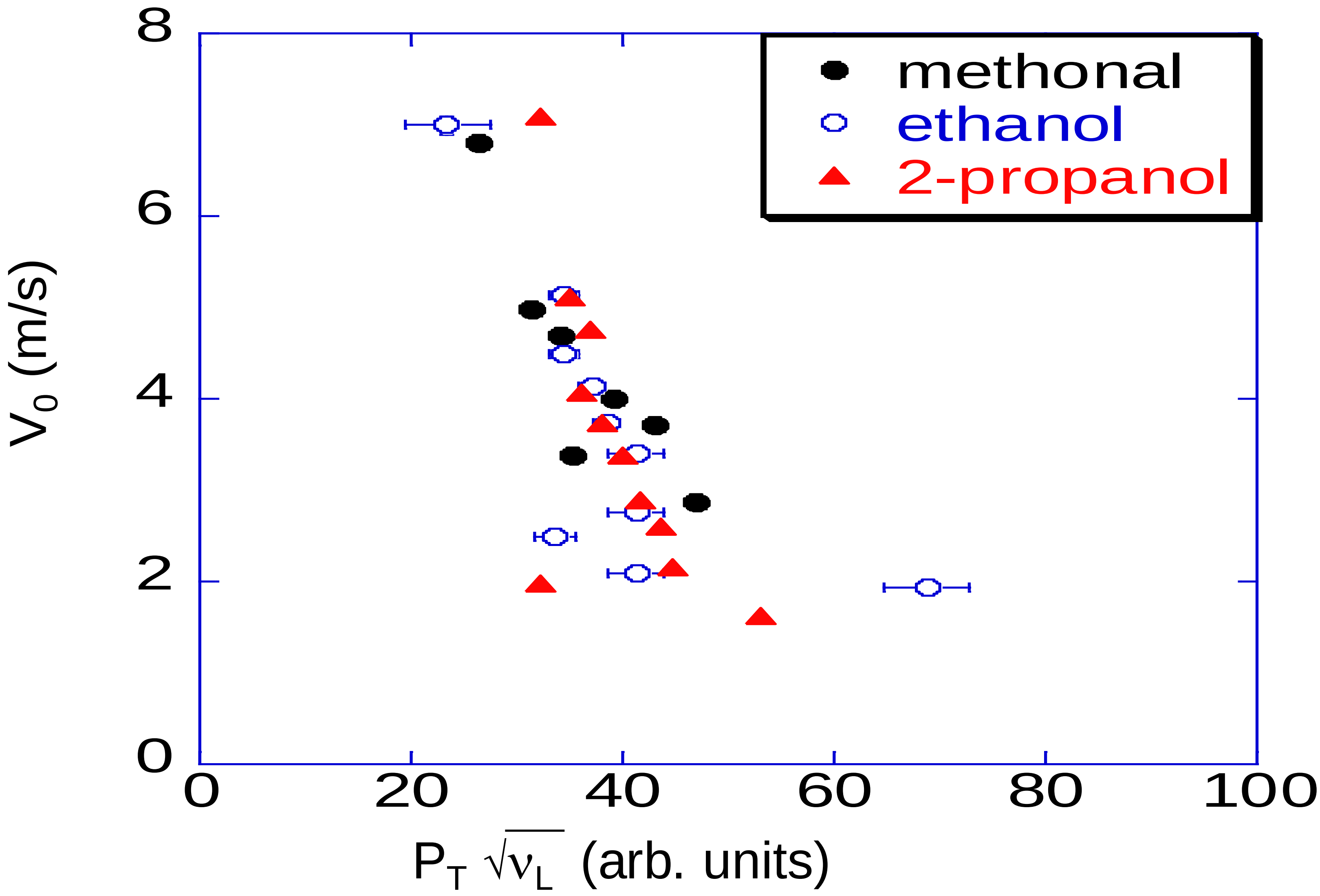
2 snapoffs:

Top
Bottom

Xiangdong Shi
Michael Brenner

As it evaporates, what remains?

Data collapse of different liquids
in high-velocity regime in air



Physical constants of gases

gas	He	Air	Kr	SF_6
Molecular Weight (Dalton)	4	29	83.8	146
Dynamic viscosity (μPa s)	20	18.6	25.6	15.3