Everything you always wanted to know about
non-Abelian quantum Hall states
but were afraid to ask

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colloquium Stockholm - January 15, 2009

physics ...

I'm astounded by people who want to 'know' the universe when it's hard enough to find your way around Chinatown.

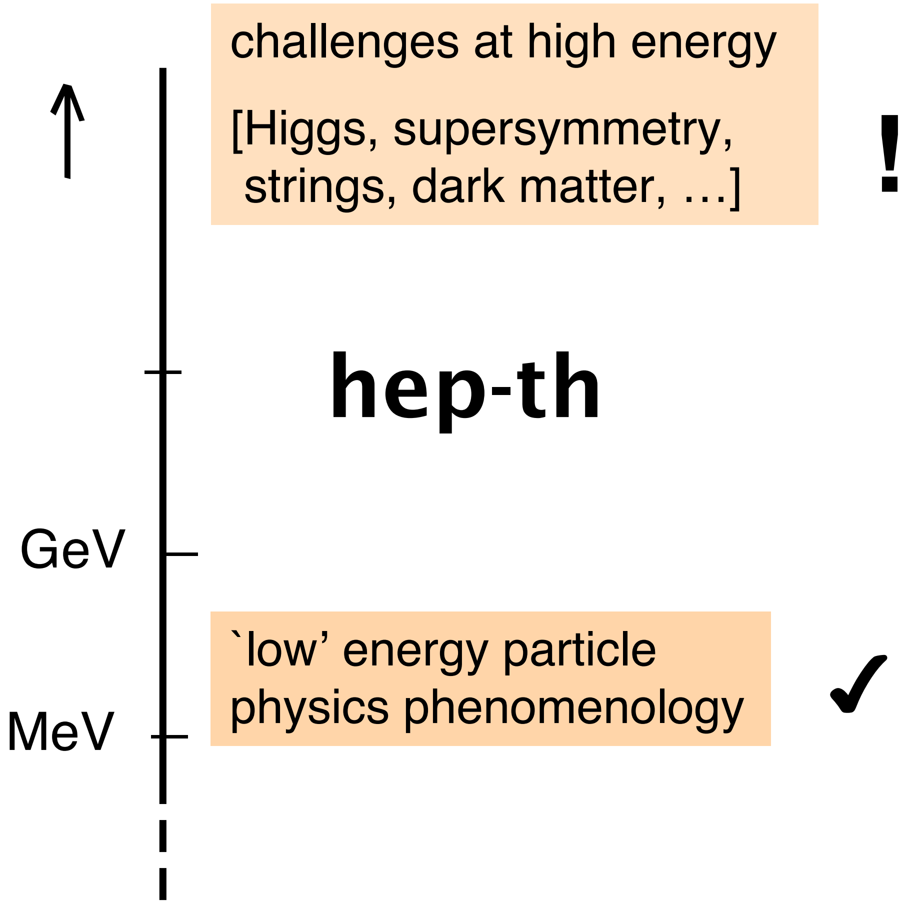
Woody Allen

physics ...

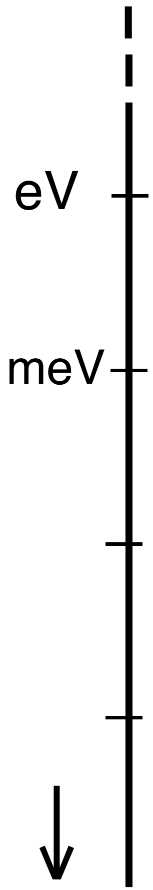
I am at two with Nature

Woody Allen

A hierarchy of scales



A hierarchy of scales



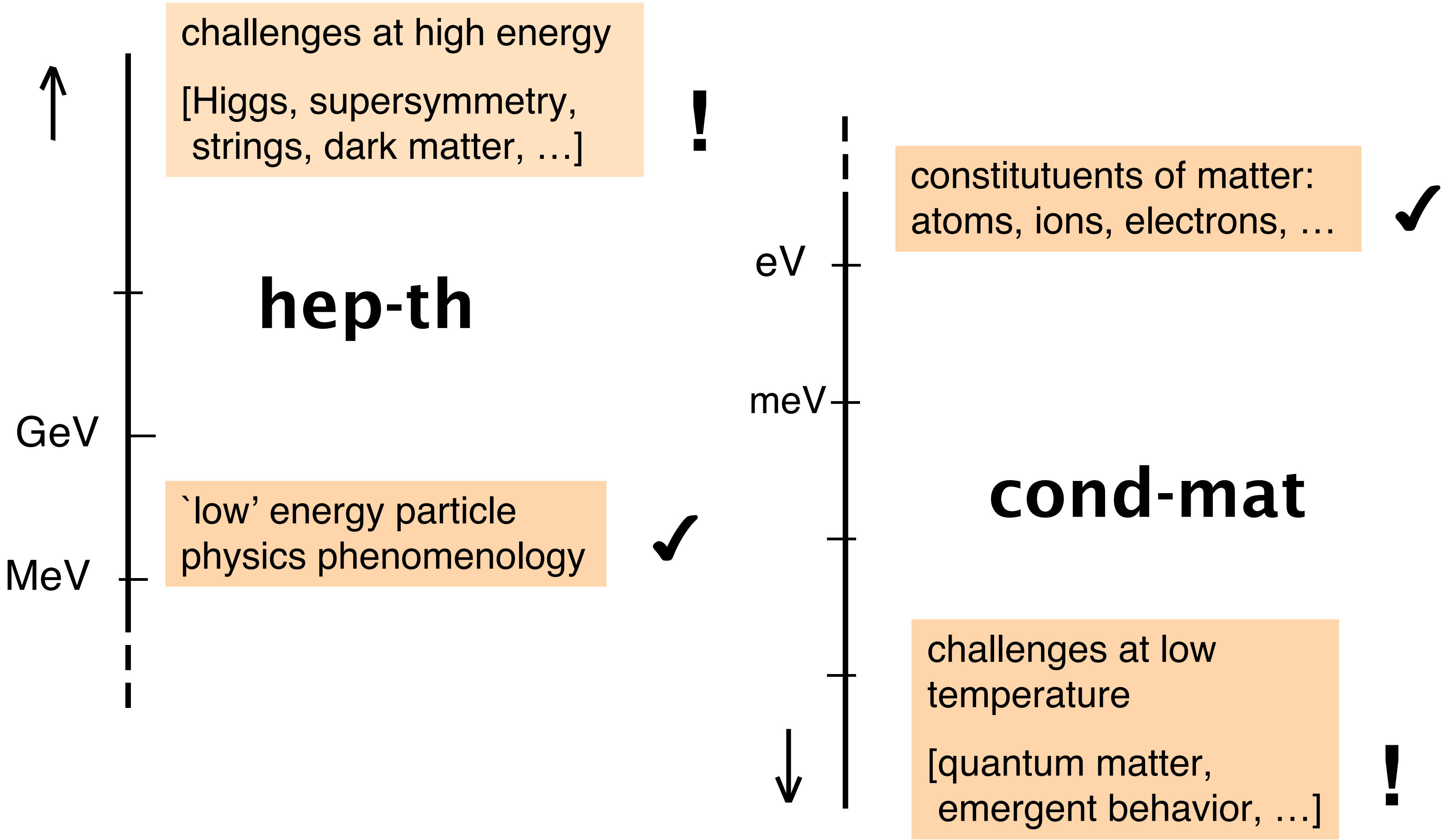
constitutuents of matter:
atoms, ions, electrons, … ✓

cond-mat

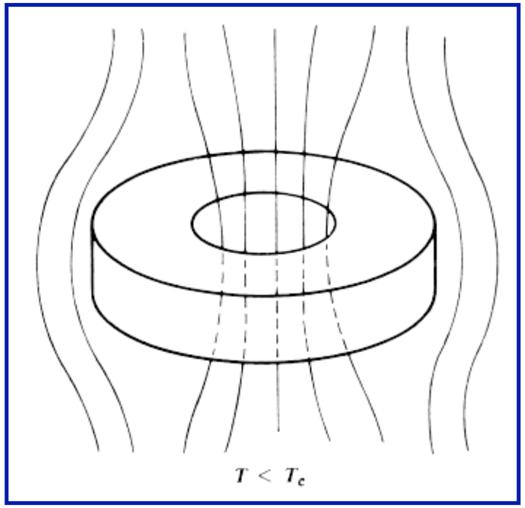
challenges at low temperature

[quantum matter,
emergent behavior, …] !

A hierarchy of scales

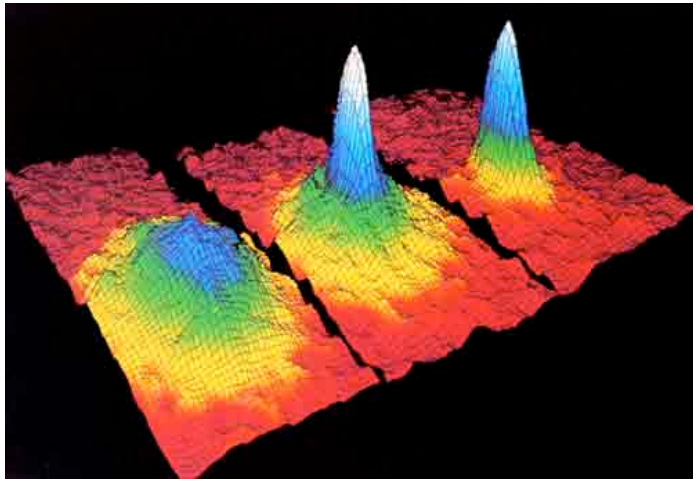


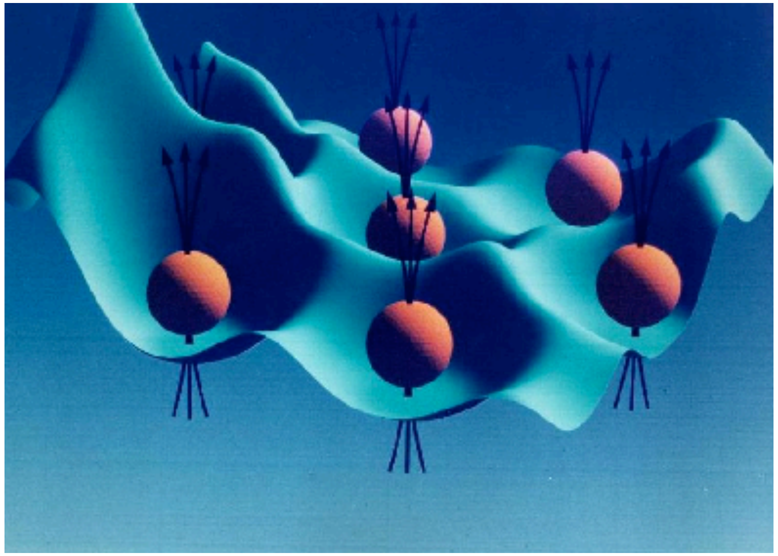
low-temperature quantum matter



- superconductor
  (condensate of paired electrons)

- BEC of cold atoms





- fractional quantum Hall state
  (liquid of correlated electrons)

low-temperature quantum matter

Today:

- quantum matter (electrons or cold atoms) in the lowest Landau level (LLL)
- elementary excitations
- non-Abelian quantum Hall states

`low-energy particle physics in the LLL'

non-Abelian quantum Hall states

What are nAqH states?

What is so special about them?

How can we detect them?

Why the name?

Can they be useful?

When will we know if they are for real?

non-Abelian quantum Hall states

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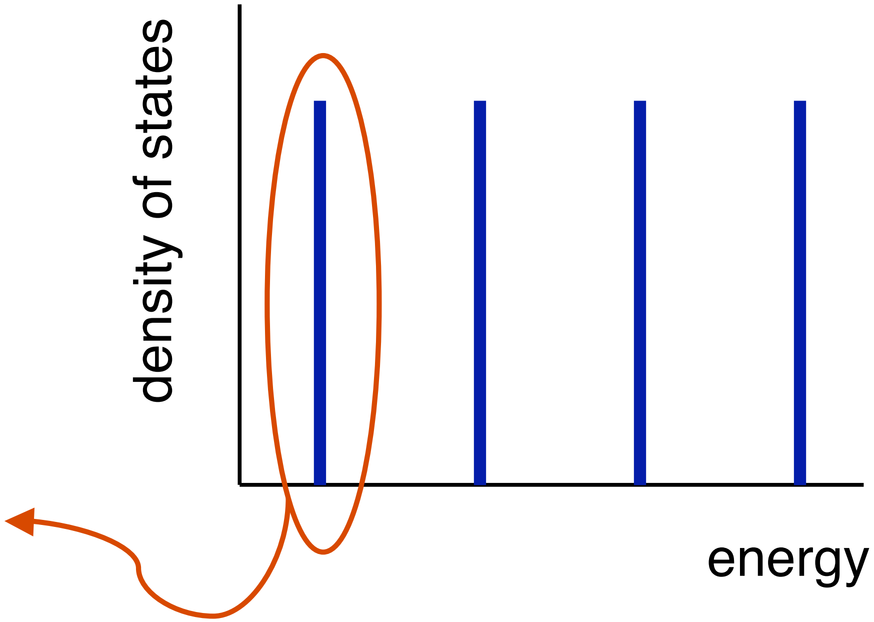
life in the Lowest Landau Level

LLL:

degenerate

single particle states

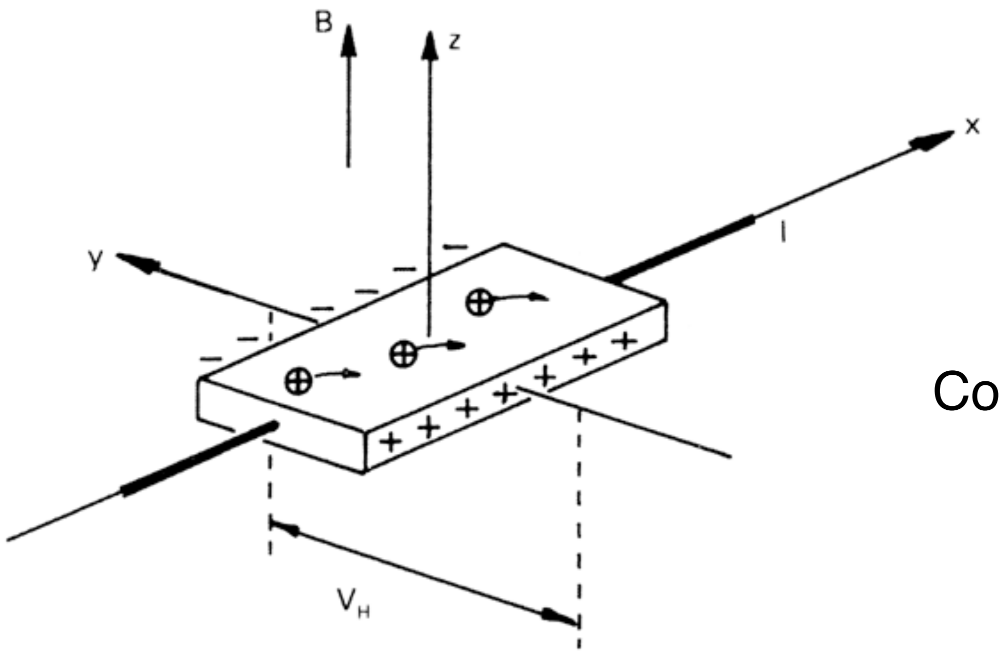


(1) charged particles in 2D with strong perpendicular magnetic field

(2) rotating bosons in the limit where rotation frequency Ω approaches the trap frequency $\omega_\perp$

life in the Lowest Landau Level

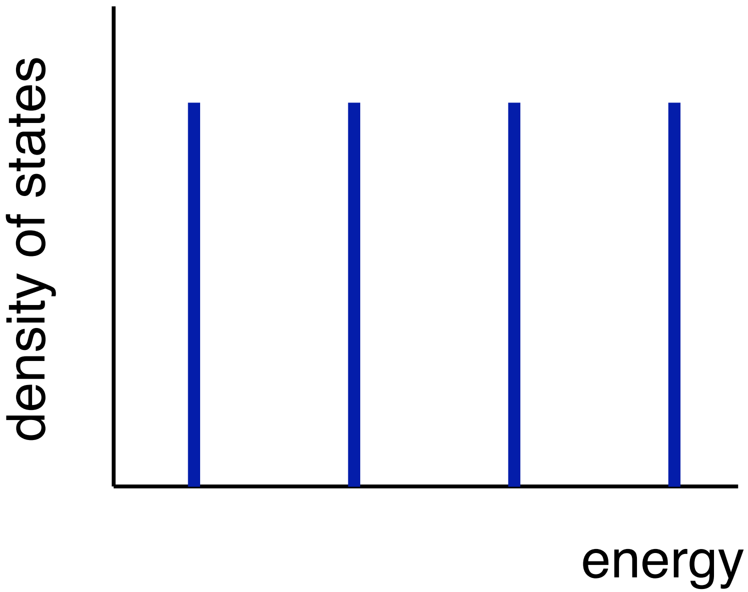
electrons under quantum Hall effect conditions



Correlated electrons

- in two dimensions
- at low temperatures
- in strong magnetic field

life in the Lowest Landau Level



In quantum mechanics, 2D electrons in a magnetic field B have LL energies

$$E = (n + \tfrac{1}{2})\,\hbar\omega_c$$

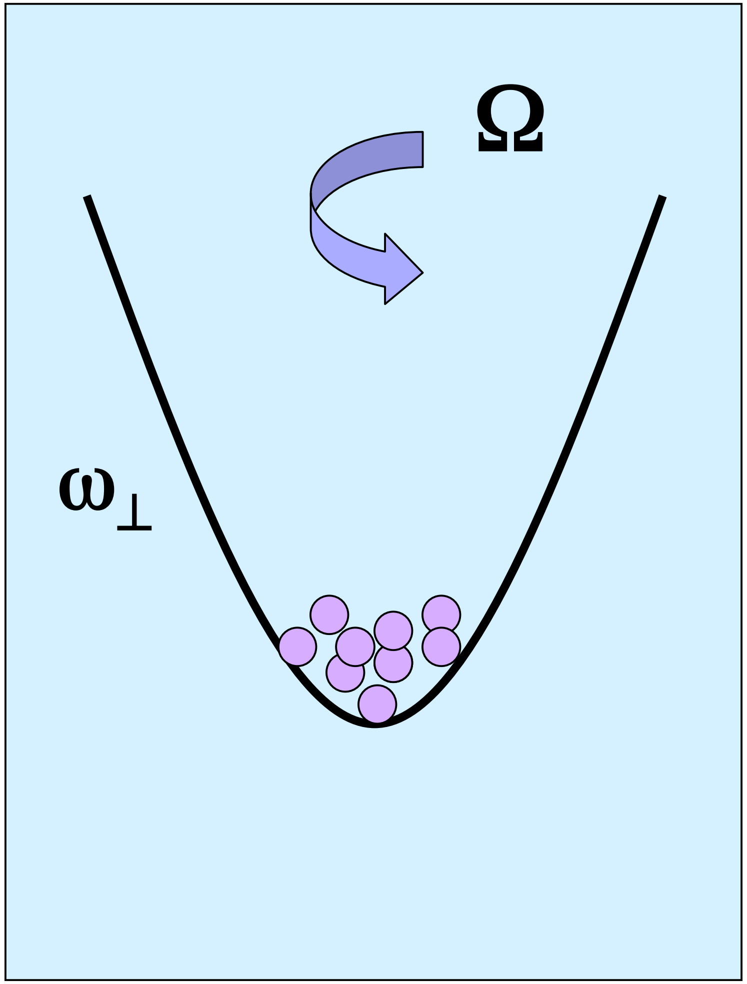
Lowest Landau Level (LLL): analytic functions, $m \geq 0$,

$$\Psi_m(z) = z^m \exp\left(-\frac{|z|^2}{4l^2}\right)$$

localized near radius $r_m = 2l\sqrt{m}$

life in the Lowest Landau Level

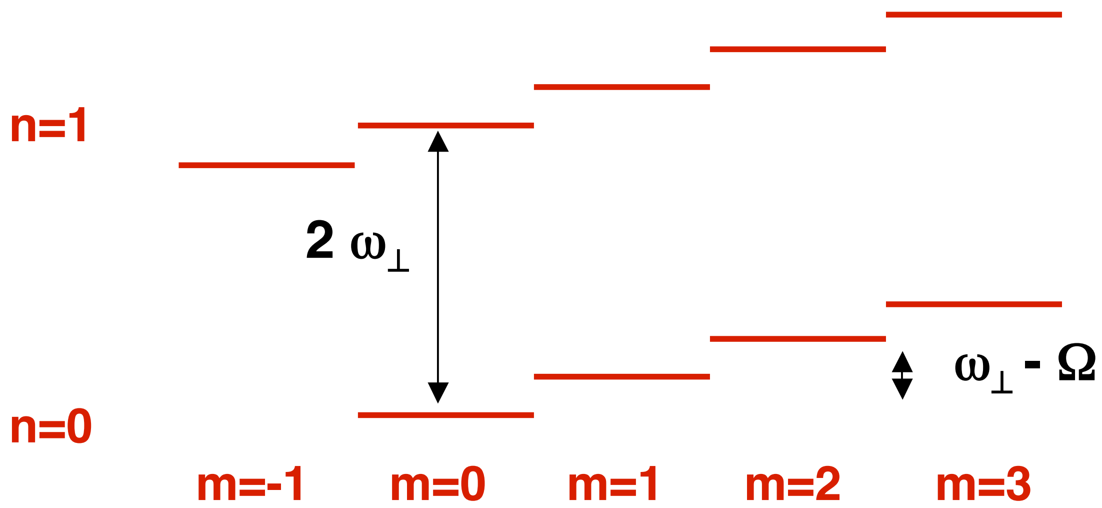
rapidly rotating bosons



LLL approached in the limit where the rotation frequency Ω approaches the trap frequency $\omega_\perp$

life in the Lowest Landau Level

single particle states for free particles in 2D harmonic trap
(frequency $\omega_\perp$) rotating at frequency Ω *(n≥0, m≥-n)*



$$\omega_\perp - \Omega \ll \omega_\perp$$

Lowest Landau Level (LLL): *n=0, m≥0* :

$$\Psi_m(z) = z^m \exp\left(-\frac{|z|^2}{4l^2}\right)$$

of energy $E=m(\omega_\perp - \Omega)$

rotating bosons in the LLL

LLL model for rotating bosons

N bosons occupying LLL orbitals

$$\Psi_m, \quad m=0,1,2,\ldots,$$

with angular momentum $L=\sum m N_m$,

and Hamiltonian

$$H = (\omega_\perp - \Omega)\, L + g \sum_{i<j=1}^{N} \delta\,(r_i - r_j)$$

rotating bosons in the LLL

BEC and vortex and vortex lattice

- angular momentum $L=0$: all bosons settle in the in the $m=0$ orbital and form a **BEC**

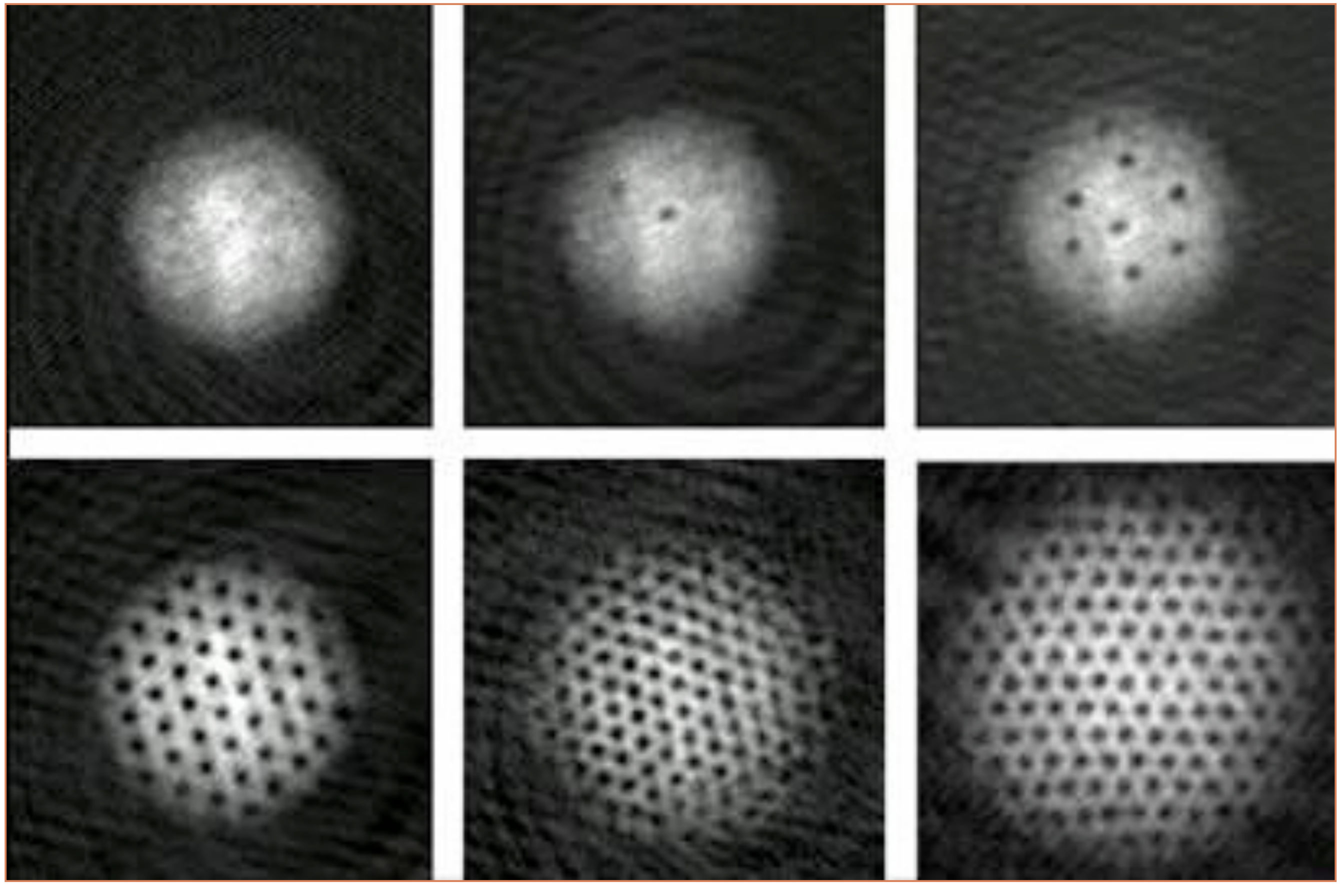
- at angular momentum $L=N$, the exact groundstate is

$$\Psi_{\text{Vortex}} = \prod_i (z_i - z_c) \exp\left(-\frac{\sum_i |z_i|^2}{4l^2}\right)$$

with $z_c = (z_1+z_2+\ldots)/N$, which is a **vortex**

- still higher angular momentum: **vortex lattice**

rotating bosons in the LLL



formation of vortices and vortex lattice in rotating trapped Rb atoms

JILA, 2003

rotating bosons in the LLL

bosonic Laughlin state

• a configuration with vanishing interaction energy becomes possible at ultra-fast rotation, $L=N(N-1)$

$$\Psi_{\text{Laughlin}} = \prod_{i<j} (z_i - z_j)^2 \exp\left(-\frac{\sum_i |z_i|^2}{4l^2}\right)$$

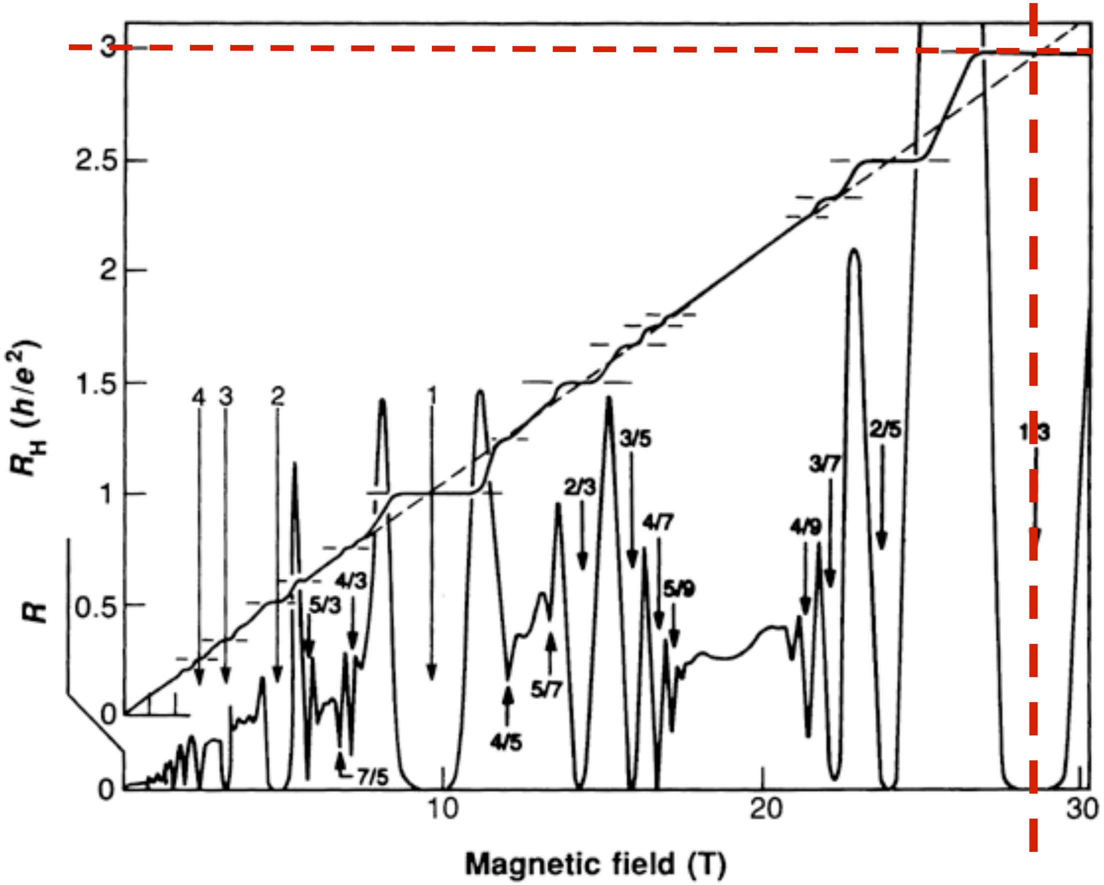
It describes a quantum liquid state with on average one particle per two vortices: $\nu = n_b / n_V = 1/2$

[not yet seen in experiments …]

electrons in the LLL

Plateaus in Hall resistance indicate incompressible quantum liquids at

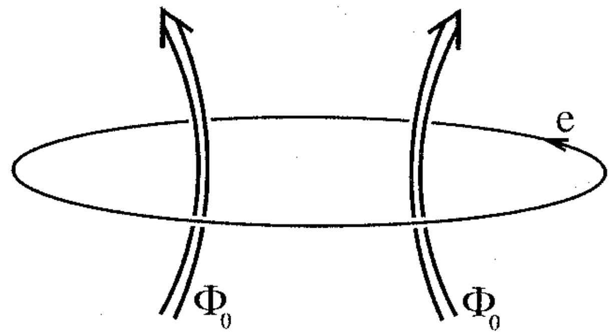
$\nu = n_e / n_\Phi = 1/3, 2/5, 3/7, \ldots$



For $\nu=1/3$ Laughlin proposed the wavefunction

$$\Psi_{\text{Laughlin}} = \prod_{i<j} (z_i - z_j)^3 \exp\left(-\frac{\sum_i |z_i|^2}{4l^2}\right)$$

electrons in the LLL



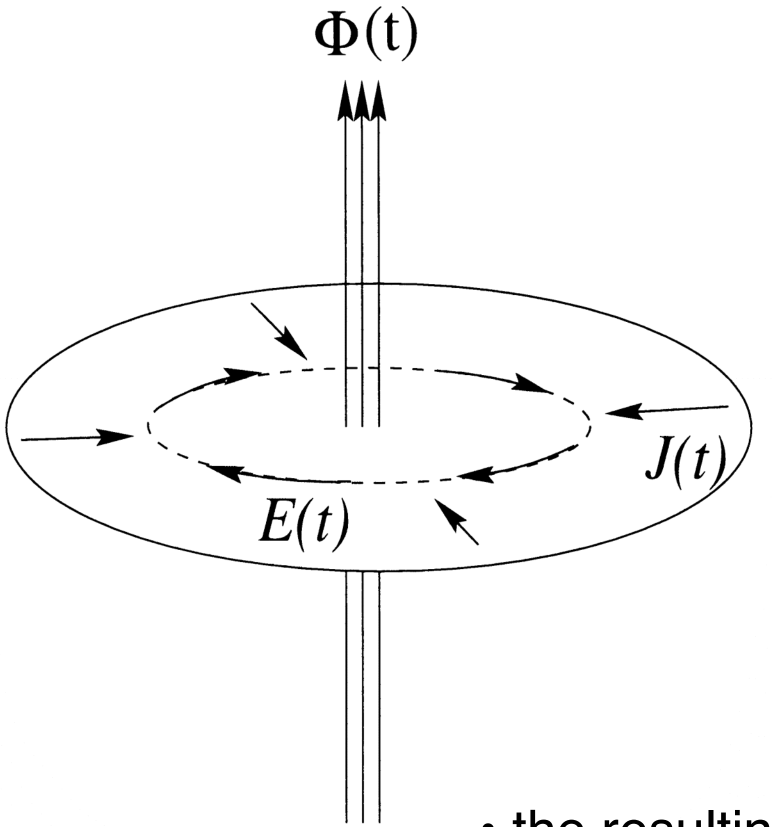
Composite Fermions (CF)

- CF obtained by attaching 2 flux quanta to each electron
- remaining electrons see reduced magnetic field; if these fill *m* Landau levels, the filling of the electrons is found to be

$$\nu = m/(2m+1) = 1/3,\ 2/5,\ 3/7,\ \ldots$$

[odd-denominator rule]

quasi-holes over Laughlin's state

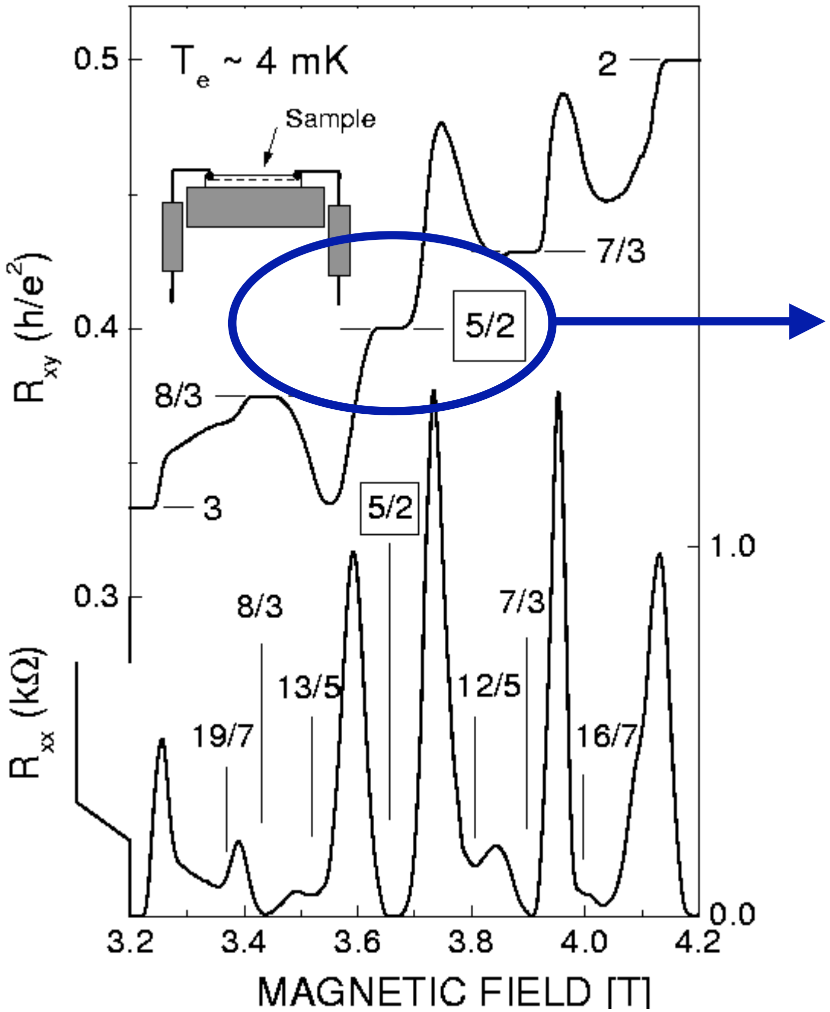


- fundamental excitation over $\nu=1/m$ Laughlin state associated to **insertion of a flux quantum Φ_0**

- the resulting quasi-hole carries an electric charge

$$q = \sigma_{\text{Hall}}\, \Phi_0 = \nu\, e^2/h\; h/e = \nu\, e = e/m$$

beyond Laughlin and CF

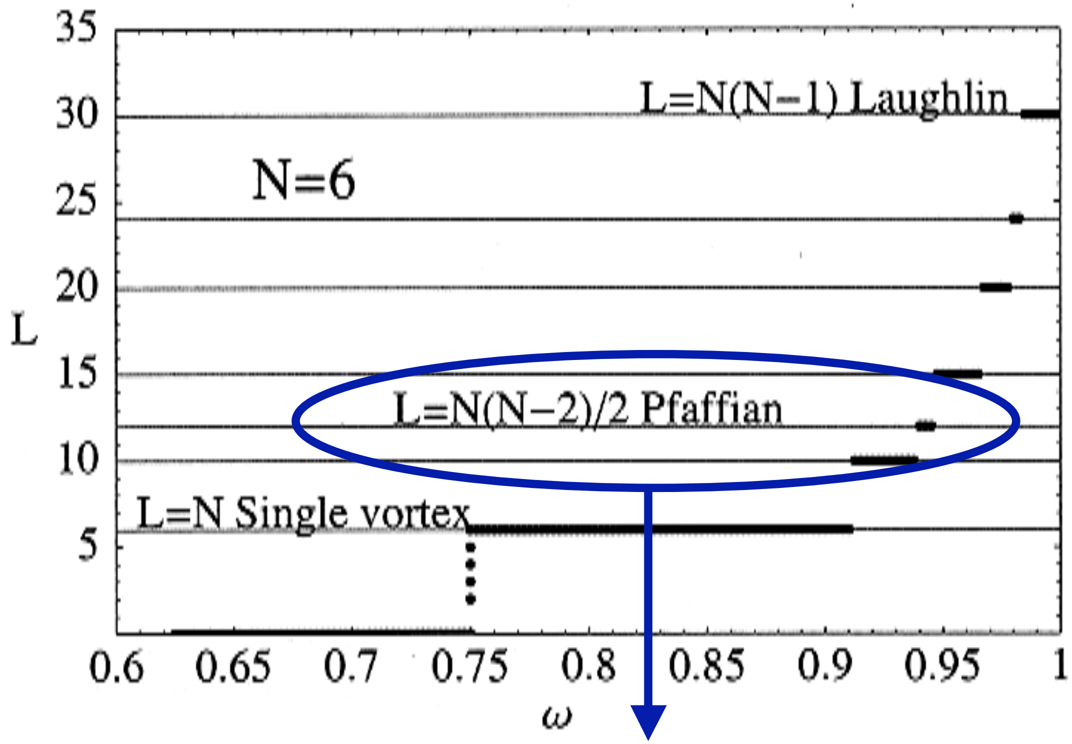


experimental findings:

- quantum Hall effect for half-filled 2nd Landau level, violating the odd-denominator rule
- spin-polarized electrons
- Fermi liquid effects at higher temperatures

Pan et al (1997, 2001)
Willlett et al (2001)

beyond Laughlin and CF



Wilkin-Gunn 1999

Cooper-Wilkin-Gunn 2001

numerical findings:

• quantum Hall states other than Laughlin identified in exact diagonalization studies of rotating bosons in the LLL

Moore-Read states

Moore-Read 1991 :

$$\Psi_{\mathrm{MR}}(z_1,\ldots,z_N) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^{M+1}\exp\left(-\frac{\sum_i |z_i|^2}{4l^2}\right)$$

M=0: bosonic qH state proposed for rotating bosons at $\nu = 1$

M=1: fermionic qH state proposed for half-filled 2nd LL, $\nu = 5/2$

real space BCS wavefunction describing (chiral) p-wave pairing of composite fermions:

MR state <-> paired quantum Hall state

non-Abelian quantum Hall states

What are nAqH states?

nAqH states:

quantum Hall states with pairing or clustering

- spin-polarized electrons, pairing
  \`Pfaffian state' (Moore-Read 1991)
- spin-polarized electrons, order-k clustering
  \`parafermion states' (Read-Rezayi 1999)
- spin-singlet electrons, order-k clustering
  \`NASS states' (Ardonne-S 1999)

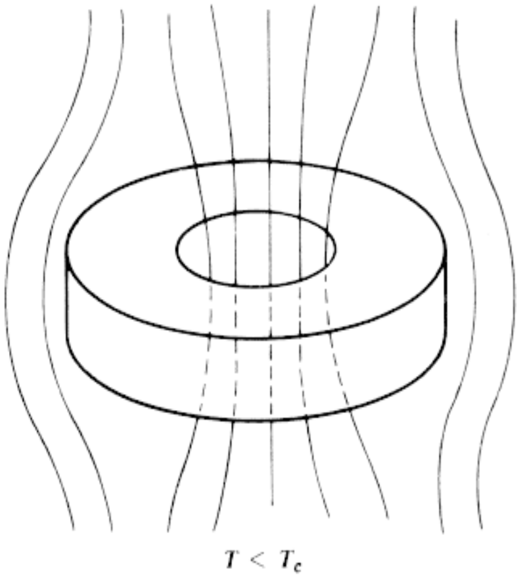
non-Abelian quantum Hall states

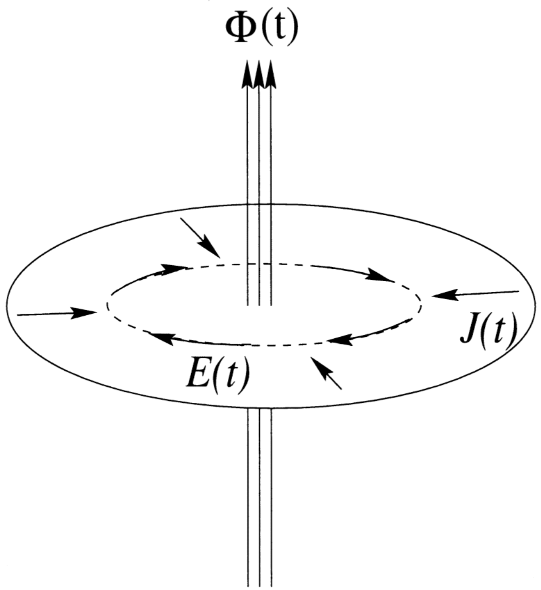
What is so special about nAqH states?

quasi-hole excitations over nAqH states:

- due to pairing / clustering, there is a break-up of the Laughlin quasi-hole
- a collection of fundamental quasi-holes forms a multi-dimensional space

quasi-holes over nAqH states





the fundamental flux-quantum through a hole in a *k*-clustered state is Φ_0 / k --> fundamental particles cut into *k* pieces

quasi-holes over nAqH states

[MR state at ν = *1/2*]

- the Laughlin quasi-hole (charge *e/2*) is cut in half: the fundamental quasi-holes have charge

$$q = e/4$$

the dimension of the internal space for *n* quasi-holes is

$$D_n = 2^{(n-2)/2}$$

quasi-holes over nAqH states

[k=3 RR and k=2 NASS states]

• the dimension of the internal space for *n* quasi-holes is

$$D_n = \mathrm{Fibonacci}_n = 1,\ 1,\ 2,\ 3,\ 5,..$$

[Fibonacci anyons]

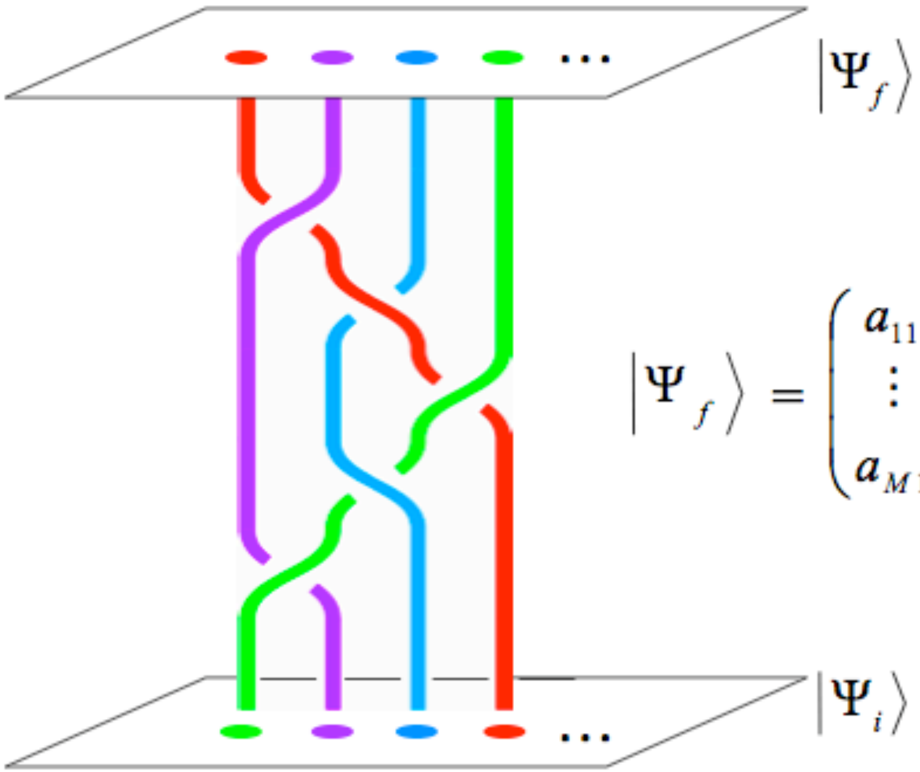
non-Abelian quantum Hall states

Why the name?

Braiding of quasi-hole excitations:

- braiding two quasi-holes around one another induces matrix acting on the state vector
- successive braidings do not commute

braiding of quasi-holes



$$|\Psi_f\rangle = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{pmatrix} |\Psi_i\rangle$$

figure N. Bonesteel

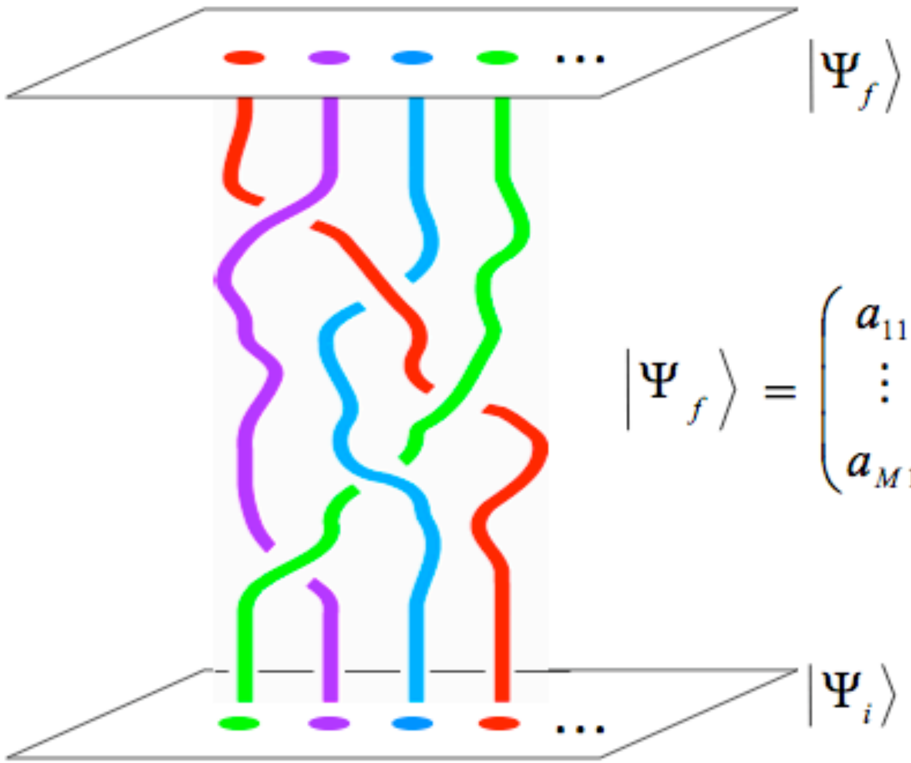
non-Abelian quantum Hall states

Can nAqH states be useful?

Topological Quantum Computation

- state vector of collection of quasi-holes viewed as qu-bit or quantum register
- information stored as `quantum knot' --> topological protection against decoherence
- logical gates implemented by braiding of quasi-holes

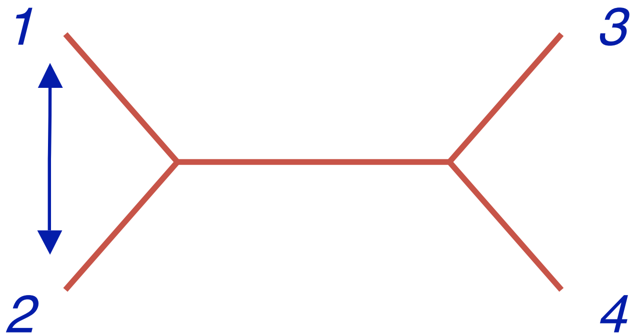
topological protection



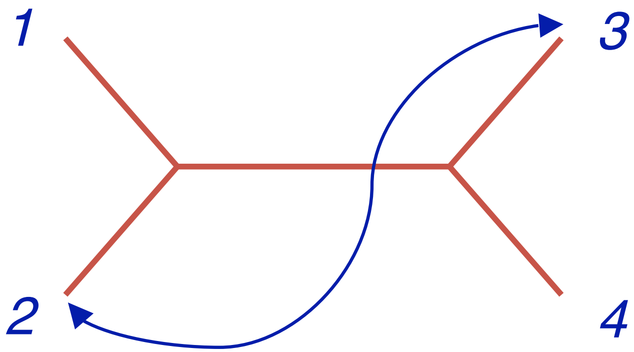
$$|\Psi_f\rangle = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{pmatrix} |\Psi_i\rangle$$

logical operations via braiding

braiding *n = 4* Fibonacci particles



$$U_{1\leftrightarrow 2} = \sigma_1 = \begin{pmatrix} (-1)^{4/5} & 0 \\ 0 & (-1)^{-3/5} \end{pmatrix}$$



$$U_{2\leftrightarrow 3} = \sigma_2 = \begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

$$\tau = \frac{1}{2}\left(\sqrt{5} - 1\right)$$

logical operations via braiding

quantum gates with Fibonacci anyons

with well-chosen iterations of σ_1 and σ_2, logical gates can be approximated to any desired precision!

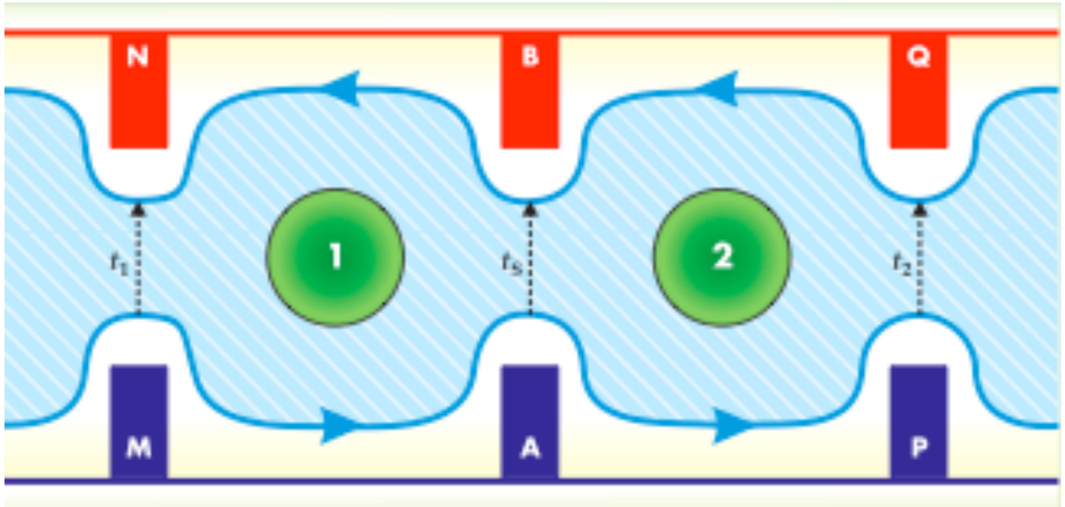
$$\sigma_1^{-2}\sigma_2^{-4}\sigma_1^{4}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{2}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{2}\sigma_2^{-4}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{-2} \approx \left(\begin{array}{cc|c} 0 & i & 0 \\ i & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

figure: NOT gate with accuracy better than 10^{-3}

Bonesteel et al, 2005

topological quantum computation

Fibonacci anyon quantum Hall quantum computer



←

nAqH state
supporting
Fibonacci anyons

das Sarma-Nayak-Freedman 2005

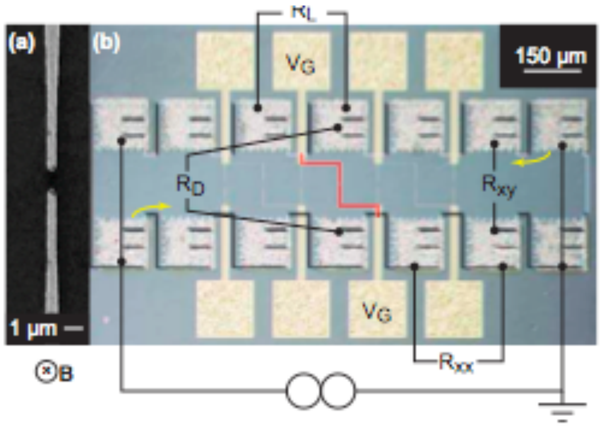
non-Abelian quantum Hall states

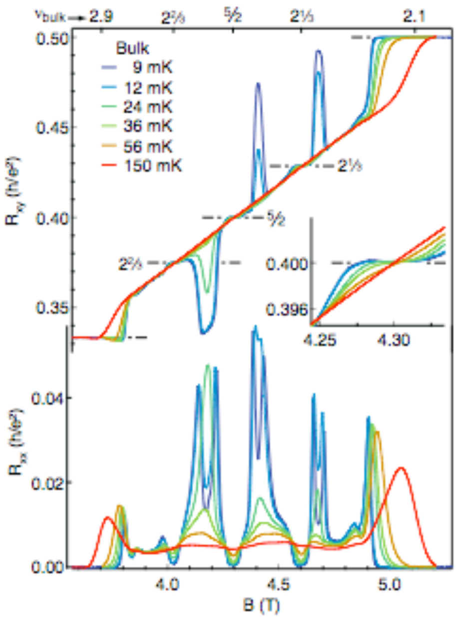
How can nAqH states be detected?

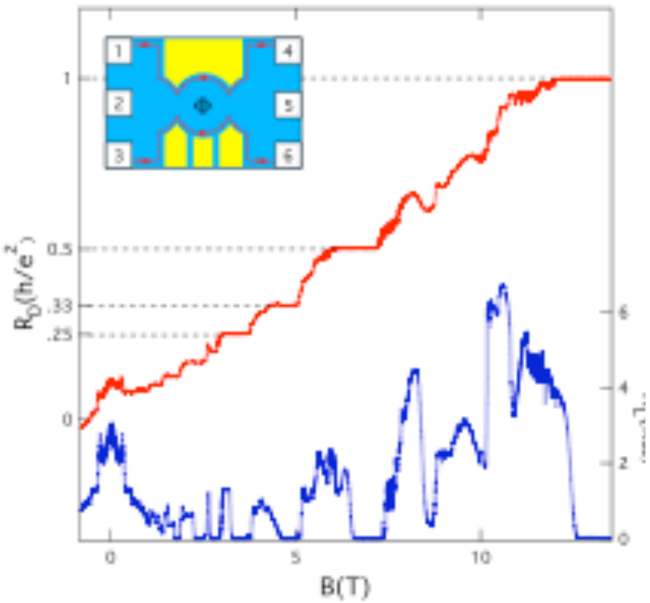
experimental signatures

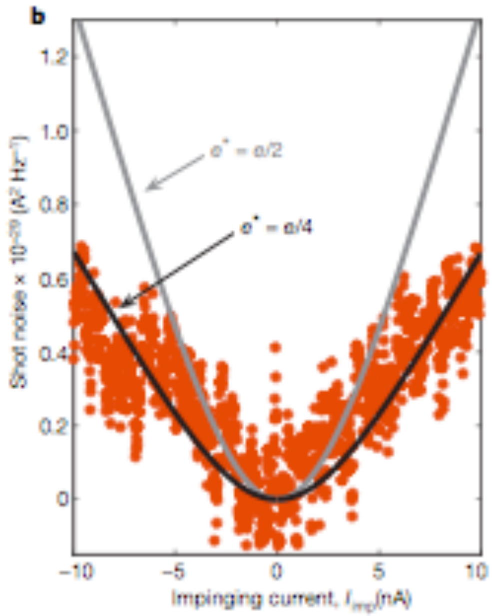
- quasi-particle charge
- interferometry with double point-contact
- Coulomb blockade signatures
- …

$\nu = 5/2$: point contacts and interferometers









recent experiments on $\nu = 5/2$ qH state with single point contact [group Marcus 2007, group Heiblum 2007] and double point contact [group Kang 2007, group Willett 2008]

non-Abelian quantum Hall states

When will we know if nAqH states are for real?

Recent progress

- quasi-particle charge: $q{=}e/4$ [Heiblum] ✔
- tunneling characteristic at point-contact [Marcus] ✔
- interferometry [Willett] **!?**
- …

Everything … about nAqH states …

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When will we know if they are for real?

Your questions …

acknowledgements

E. Ardonne, N. Read, E. Rezayi,

[wavefunctions, statistics, …]

N.R. Cooper, N. Read, F. van Lankvelt, J.W. Reijnders,

[rotating bosons]

E. Fradkin, C. Nayak

[effective field theories]

E. Grosfeld, R. Ilan, A. Stern

[experimental signatures]